The advantages of the Eta model dynamics Fedor Mesinger

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#### Eta dynamics: What is being done?

- Gravity-inertia wave terms, B/E grid: forward-backward scheme that

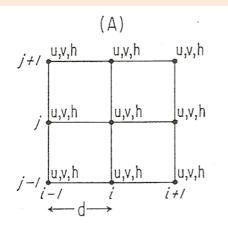
   (1) avoids the time computational mode of the leapfrog scheme, and is
   neutral with time steps twice leapfrog;
- (2) modified to enable propagation of a height point perturbation to its nearest-neighbor height points/suppress space computational mode;
- Split-explicit time differencing (very efficient);
- Horizontal Arakawa advection that conserves energy and C-grid enstrophy, on the B/E grid, in space differencing (Janjić 1984);
- Conservation of energy in transformations between the kinetic and potential energy, in space differencing;
- Finite-volume vertical advection of dynamic variables (v, T)
- Nonhydrostatic option;

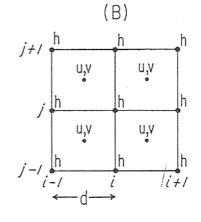
• The (cut-cell) eta vertical coordinate, ensuring hydrostatically consistent calculation of the pressure gradient ("second") term of the pressure-gradient force (PGF);

Before we get into some of these: To solve our equations we use values at grid points:

we need a horizontal grid, and a vertical grid horizontal Primitive equations

# Four possible square grids:





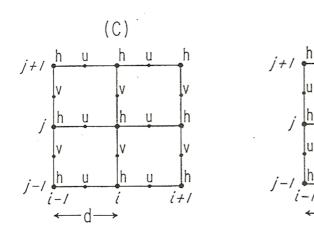
(D)

h

٧

h

i+1



(E)

,u,v

 $j \neq \frac{j}{2}$   $\frac{U,V}{V}$ 

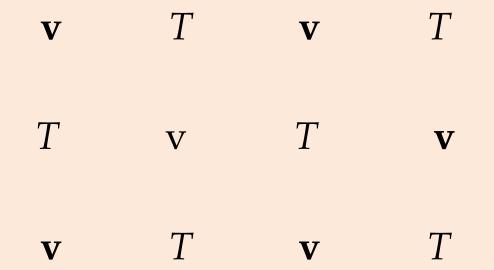
Note: E grid is same as B but rotated 45°. Thus, often: E/B, or B/E



FIG. 3. Spatial distributions of the dependent variables on a square grid.

What are the values at the grid points?

With "primitive equations", and the E grid horizontal grid consists of  $\mathbf{v}(u,v)$ , and *T* points:



Two main possibilities: values of continuous fields, taken at points, or averages over grid cells

Averages over grid cells: Reynolds averages

## This view taken in the Eta dynamics, "Finite-volume" approach

With this approach formal, Taylor series type order of accuracy, has a questionable meaning

In slide #2, eight features have been listed. Their purposes are different

- Some, to increase accuracy by avoiding recognized possible errors,
- Others, same, but by avoiding "computational modes",
- Still others, same, but by maintaining integral properties,
- Yet others, to increase computational efficiency, along with some of the above

#### Eta dynamics: What is being done?

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#### Eta dynamics: What is being done?

Gravity wave terms, on the B/E grid: forward-backward scheme that

 (1) avoids the time computational mode of the leapfrog scheme, and is
 neutral Perhaps, the most; unique
 (2) modified to enable propagation of a height point perturbation to its
 nearest-And/orgmost beneficial:putational
 mode;

Split-explicit time differencing (very efficient)

• Horizontal Arakawa advection that conserves energy and C-grid enstrophy, on the B/E grid, in space differencing (Janjić 1984);

• Finite-volume vertical advection of dynamic variables (v, T)

Nonhydrostatic option;

• The (cut-cell) eta vertical coordinate, ensuring hydrostatically consistent calculation of the pressure gradient ("second") term of the pressure-gradient force (PGF);

#### Arakawa horizontal advection schemes

The first "general circulation" experiment:

Phillips, N. A., 1956. *The general circulation of the atmosphere: a numerical experiment*. Quart. J. Roy. Meteor. Soc., 82, 123-164.

A problem: features / energy was accumulating at small scales Arakawa energy / enstrophy conserving schemes address

Nondivergent vorticity equation, Arakawa (1966):

$$\frac{\partial \zeta}{\partial t} + \mathbf{v} \cdot \nabla \zeta = 0; \quad \zeta = \nabla^2 \psi, \qquad (7.1)$$

where the velocity v is assumed to be nondivergent, that is

$$\mathbf{v} = \mathbf{k} \times \nabla \boldsymbol{\psi} \,. \tag{7.2}$$

Substituting this into (7.1) we obtain

$$\frac{\partial}{\partial t} \nabla^2 \psi = J \left( \nabla^2 \psi, \psi \right). \tag{7.3}$$

 $\psi = \sum_{n} \psi_{n}$ , (e.g. Courant and Hilbert, 1953, p. 369)

where the functions  $\psi_n$  are eigenfunctions of the Helmholtz equation

$$\nabla^2 \psi_n + \lambda_n^2 \psi_n = 0. \qquad (7.5)$$

The parameters  $\lambda_n$  are known as the generalized <u>wavenumbers</u> of the components  $\psi_n$ .

As an example, let A be a rectangular region with sides  $L_x$ ,  $L_y$ . For boundary conditions assume that the stream function is periodic in A: with period  $L_x$  and is zero along the lower and upper boundary. Then we can write the stream function

$$\Psi = \sum_{n_1, n_2} \left( a_{n_1 n_2} \cos \frac{2\pi n_1}{L_x} x + b_{n_1 n_2} \sin \frac{2\pi n_1}{L_x} x \right) \sin \frac{\pi n_2}{L_y} y.$$
(7.6)

Differentiating this we obtain  $\nabla^2 \psi_n = -\left[ \left( \frac{2\pi n_1}{L_x} \right)^2 + \left( \frac{\pi n_2}{L_y} \right)^2 \right] \psi_n \qquad \lambda_n^2 = \left( \frac{2\pi n_1}{L_x} \right)^2 + \left( \frac{\pi n_2}{L_y} \right)^2$ 

$$\overline{K} = -\frac{1}{2} \sum_{m} \sum_{n} \overline{\psi_m \nabla^2 \psi_n} = \frac{1}{2} \sum_{m} \sum_{n} \lambda_n^2 \overline{\psi_m \psi_n}$$

Since the functions  $\psi_n$  are orthogonal, that is,

 $\psi_m \psi_n = 0 \text{ for } m \neq n,$ 

We have therefore expressed the average kinetic energy in the region A as a sum of contributions of different harmonics

$$\overline{K} = \sum_{n} K_{n} , \qquad (7.8)$$
where
$$K_{n} \equiv \frac{1}{2} \lambda_{n}^{2} \overline{\psi_{n}^{2}} .$$

#### Very similar for the mean square vorticity:

$$\overline{\zeta^2} = \overline{\left(\nabla^2 \psi\right)^2}$$

can be expressed as a sum of contributions of different harmonics in a similar way. Substituting (7.4), using (7.5), and the orthogonality of the functions  $\psi_n$ , we obtain

Substituting the expression (7.9) for the kinetic energy of a component  $\psi_n$ ; we find for the average value of the *enstrophy*, half the vorticity squared,

$$K_n \equiv \frac{1}{2}\lambda_n^2 \overline{\psi_n^2}. \quad (7.9) \qquad \qquad \frac{1}{2}\overline{\zeta^2} = \sum_n \lambda_n^2 K_n. \quad (7.11)$$

Comparing this with (7.8) we see that the average wavenumber is related to average values of enstrophy and kinetic energy. Define the average wavenumber as

$$\lambda \equiv \sqrt{\sum_{n} \lambda_{n}^{2} K_{n}} / \sum_{n} K_{n} .$$

Substituting (7.11) and (7.8) we find

$$\lambda = \sqrt{\frac{1}{2}\overline{\zeta^2}/\overline{K}} \ .$$

(7.13)

(7.12) Thus: when the velocity is twodimensional and nondivergent, the average wavenumber is determined by the ratio of the average values of enstrophy and kinetic energy! From (7.13) and (7.11):

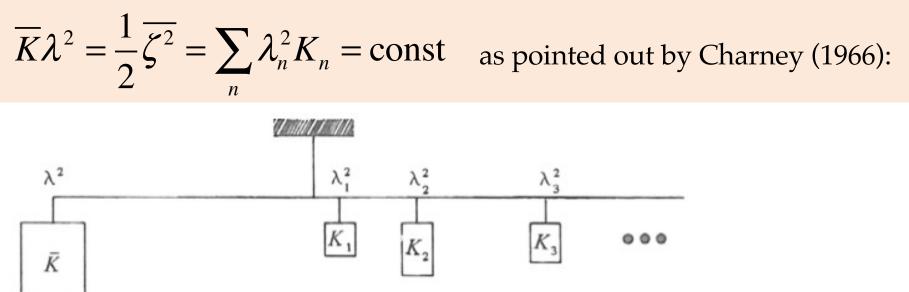


Figure 7.1 A mechanical analogy of the interchange of energy between harmonic components

From (7.13) and (7.11):

$$\overline{K}\lambda^2 = \frac{1}{2}\overline{\zeta^2} = \sum_n \lambda_n^2 K_n = \text{const}$$
 as pointed out by Charney (1966):

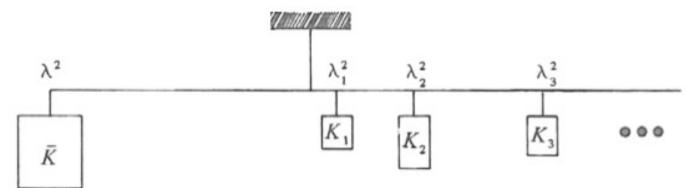
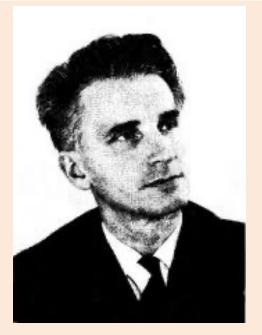


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Ragnar Fjørtoft (1913-1998)



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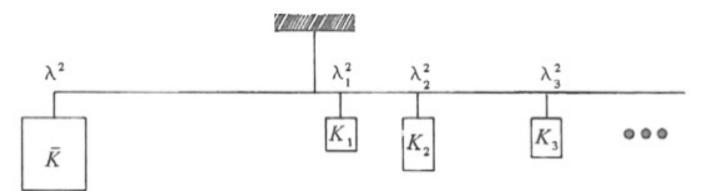
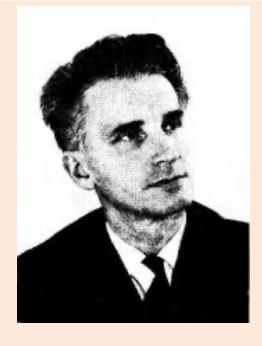


Figure 7.1 A mechanical analogy of the interchange of energy between harmonic components

Ragnar Fjørtoft (1913-1998)



Jule Charney (1917-1981)



Early NWP and general circulation (Norman Phillips !) experience has shown that numerical models have problems in behaving quite differently - energy accumulating at small scales, with catastrophic results :(

Can one reproduce this feature of the continuous equations? Akío Arakawa! (1966)

International symposium on numerical weather forecasting Oslo, March 11–16, 1963

We illustrate Arakawa's method by considering how to satisfy  $(7.17)_1$ . In our finite difference calculation it takes the form

$$(7.17)_1:$$

$$\overline{pJ(p,q)} = 0$$

$$\overline{\zeta_{ij}J_{ij}}\left(\zeta,\psi\right) = \frac{1}{N} \sum_{i,j} \zeta_{ij}J_{ij}\left(\zeta,\psi\right) = 0, \qquad (7.19)$$

$$J\left(p,q\right) = \frac{\partial p}{\partial x} \frac{\partial q}{\partial y} - \frac{\partial p}{\partial y} \frac{\partial q}{\partial x} = \frac{\partial}{\partial y} \left(q \frac{\partial p}{\partial x}\right) - \frac{\partial}{\partial x} \left(q \frac{\partial p}{\partial y}\right) =$$

$$= \frac{\partial}{\partial x} \left(p \frac{\partial q}{\partial y}\right) - \frac{\partial}{\partial y} \left(p \frac{\partial q}{\partial x}\right). \qquad (7.21a)$$

$$J^{++}\left(p,q\right) = \frac{1}{4d^2} \left[\left(p_1 - p_3\right)\left(q_2 - q_4\right) - \left(p_2 - p_4\right)\left(q_1 - q_3\right)\right], \qquad (7.21a)$$

$$J^{\times+}\left(p,q\right) = \frac{1}{4d^2} \left[q_2\left(p_5 - p_6\right) - q_4\left(p_8 - p_7\right) - q_1\left(p_5 - p_8\right) + q_3\left(p_6 - p_7\right)\right], \qquad (7.21b)$$

 $J^{+\times}$ 

$$[p,q] = \frac{1}{4d^2} \Big[ p_1(q_5 - q_8) - p_3(q_6 - q_7)$$

$$- p_2(q_5 - q_6) + p_4(q_8 - q_7) \Big].$$

$$More general: J(p,q) = \alpha J^{++} + \beta J^{\times +} + \gamma J^{+\times}$$

$$(7.22)$$

not only do all the terms in the sum (7.19) cancel, but also all the terms in the expression for the conservation of the average kinetic energy, and the average vorticity (Arakawa, 1966; Lilly, 1965). Thus, the approximation

$$J_{A} \equiv \frac{1}{3} \left( J^{++} + J^{\times +} + J^{+\times} \right), \tag{7.23}$$

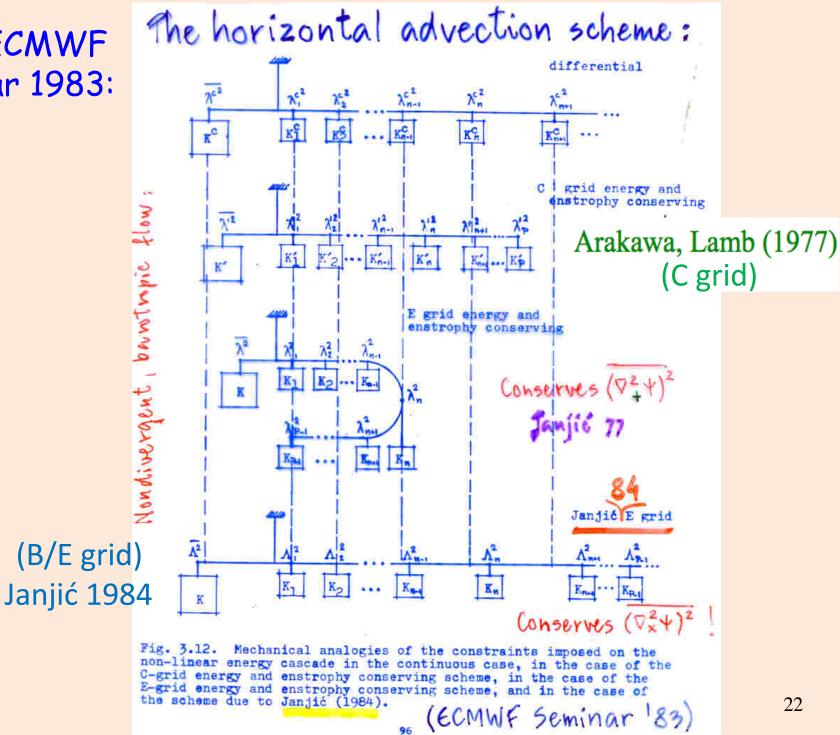
will conserve average vorticity, enstrophy and kinetic energy when used for the numerical solution of (7.3). This is more than sufficient for the prevention of nonlinear instability. The approximation (7.23) is usually called the *Arakawa Jacobian*. Arakawa has also shown how to construct an approximation of fourth order accuracy to the Jacobian, conserving these three quantities. Arakawa vorticity equation scheme transformed to the C-grid:

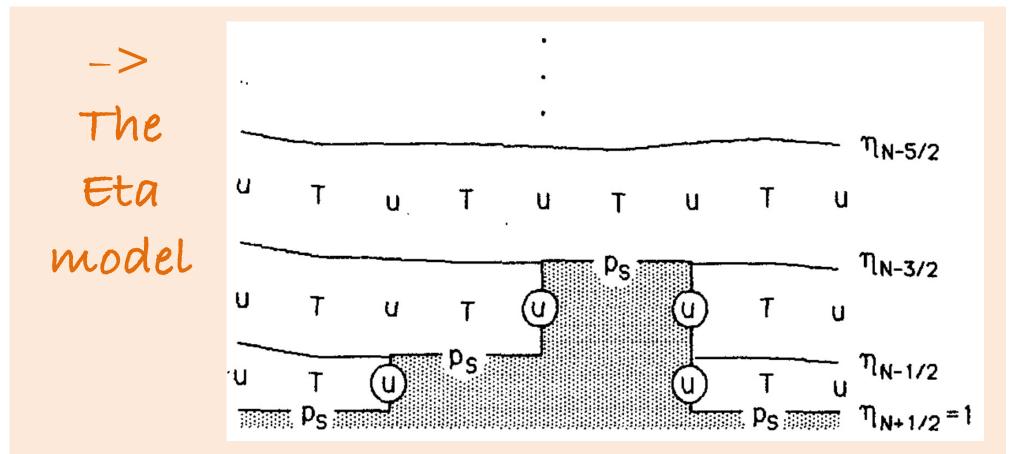
Arakawa A. and V. R. Lamb, 1977: Computational design of the basic dynamical processes of the UCLA general circulation model. *Methods in Computational Physics,* J. Chang, Ed., Academic Press, 174–264. ("The Green Book")

The C-grid Arakawa scheme transformed to the B/E-grid:

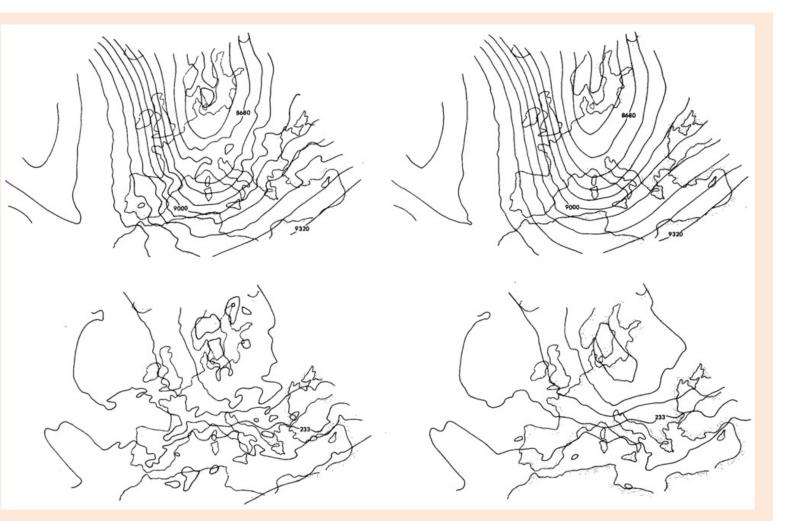
Janjić, Z. I., 1984: Nonlinear advection schemes and energy cascade on semistaggered grids. *Mon. Wea. Rev.*, **112**, 1234-1245.

#### From ECMWF Seminar 1983:





300 hPa geopotential heights (above) and temperatures (below) in a 48-h simulation using the sigma system (left) and using the eta system (right). Contour intervals are 80 m for geopotential heights, and 2.5 K for temperature. From Mesinger et al. (1988)



Eta primary regional operational model at U.S. National Meteorological Center as of March 1993. At INPE, 1996.

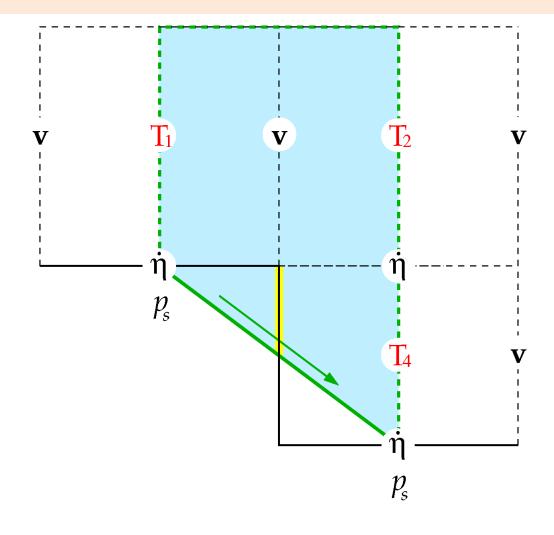
However,

• Experimental 10-km Eta did poorly a windstorm in the lee of Wasatch mountain, while a sigma system MM5 did well,

• Gallus and Klemp (2000) published experiments on flow over a bell-shaped topography. Gallus and Rančić eta coordinate model failed to simulate downstream flow, instead had the flow in the lee separate off the top of the topography Gallus and Klemp ascribed the problem to the existence of step corners of the step topography Eta, therefore:

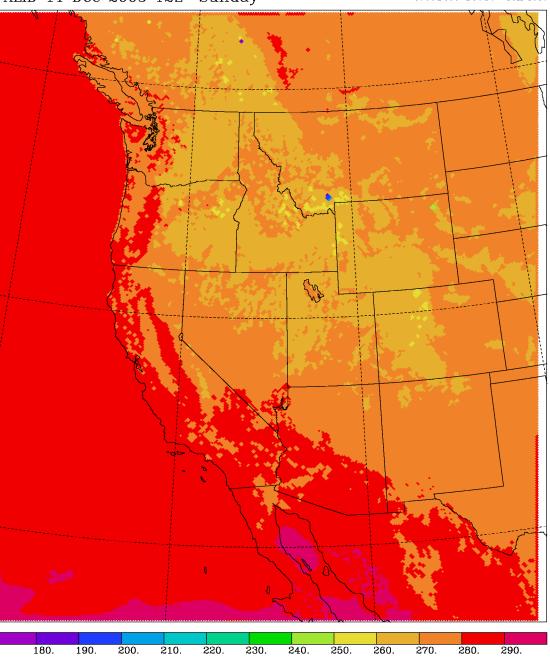
#### The sloping steps (a simple cut-cell scheme), vertical grid:

The central v box exchanges momentum, on its right side, with v boxes of two layers, and  $T_1$ box undergoes horizontal advection to  $T_2$  and vertical (slantwise) advection to  $T_4$ 



VALID 11 Dec 2005 12Z Sunday

20051209 12UTC 48h fcst



When this was coded and tested, 48-h lowest T boxes map:

| 267_42                        | 267.42 265               | 42 265 42                   | 269.42  |
|-------------------------------|--------------------------|-----------------------------|---------|
| 311,265                       | 312,264 313,2            | 314, 262                    | 315,261 |
| 42 269<br><sup>65</sup> 311.3 |                          | 263.42 267<br>313.262 314.3 |         |
| 269.42                        | 187.42 195               | 42 263 42                   | 267.42  |
| 310.264                       | 311.263 312.2            | 313,261                     | 314.260 |
| 42 267                        | 42 193 42                | 199.42 263                  | 42 263. |
| 310.3                         | <sup>263</sup> 193 42    | 312.261 313.2               | 314.2   |
| 269.42                        | 197.42 199               | 42 203 42                   | 263.42  |
| 309,263                       | 310,262 311,2            | 312,260                     | 313,259 |
| 42 267                        | 42 199 42<br>262 310,261 | 201.42 263                  | 42 259. |
| 267.42                        | 267.42 <b>201</b>        | 42 263.42                   | 263.42  |
| 308,262                       | 309,261 310.2            | 311,259                     | 312,258 |
| 42 267<br>308.3               |                          | 265.42 265<br>310,259 311.3 | 42 267. |

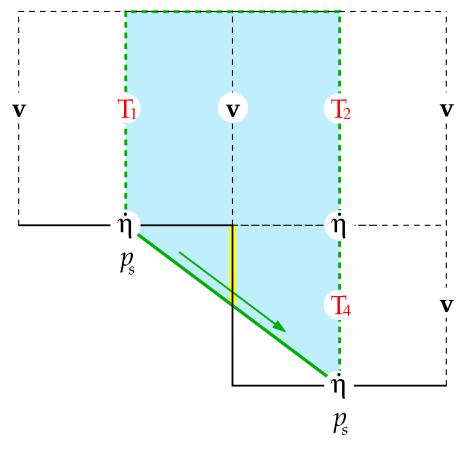
## Suspect: slantwise T advection:

standard "Lorenz-Arakawa" centered vertical advection scheme (Arakawa and Lamb 1977)

$$\frac{\partial T}{\partial t} = \dots - \frac{\partial T}{\partial \eta}^{\eta}$$
(2)

It allows a false vertical advection from below ground !! If a temperature inversion were to develop at the bottom of a basin, with a persistent upward motion, then the vertical advection contribution from the interface between the lowest T cell and the one above it would cool both cells, but for the lower of them would be the only contribution, thus tending to increase the inversion, amplifying its cooling, feeding on itself !!! In addition, this advection into the lowest cell, is physicaly wrong, no advection should exist into the lowest cell from below ground !! v T<sub>1</sub> v T<sub>2</sub> v

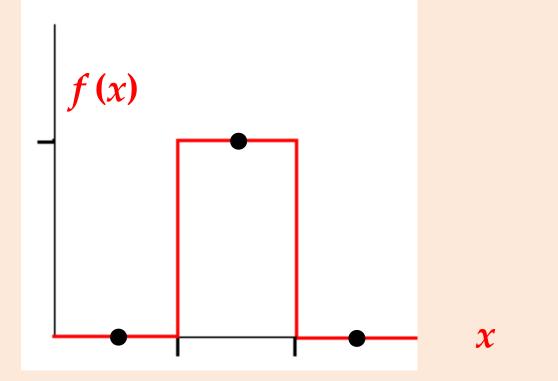
But with the finite-volume approach, with v constant inside the bluish v cell, as well as the  $T_1$  and  $T_4$  inside their cells, we can calculate how much air is crossing the yellow line and replace the wrong slantwise advection with correct T changes !!!



### Done, however

Since the problem exists in vertical advection as well, vertical advetion of v and T replaced as well !!

Piecewise linear (finite-volume) advection scheme used Consider advection of a top hat (or step) function, e.g.:



Creation of false maxima or minima using centered schemes ! Advection of moisture :(

## From Mesinger, Jovic (NCEP Office Note 2002):

#### Slope adjustment scheme

Slopes can be adjusted, but no new maxima or minima must be created. This is the first iteration. If we are not next to a minimum or maximum, we can go only half of the smaller of two sides

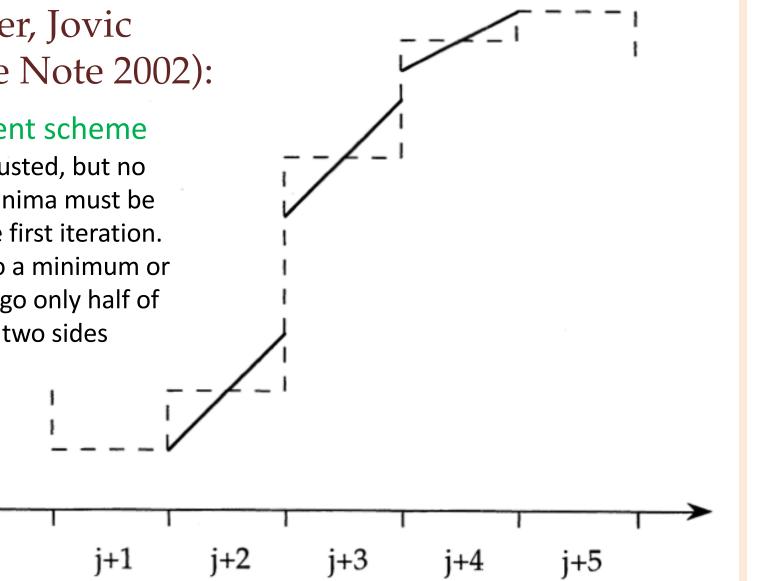
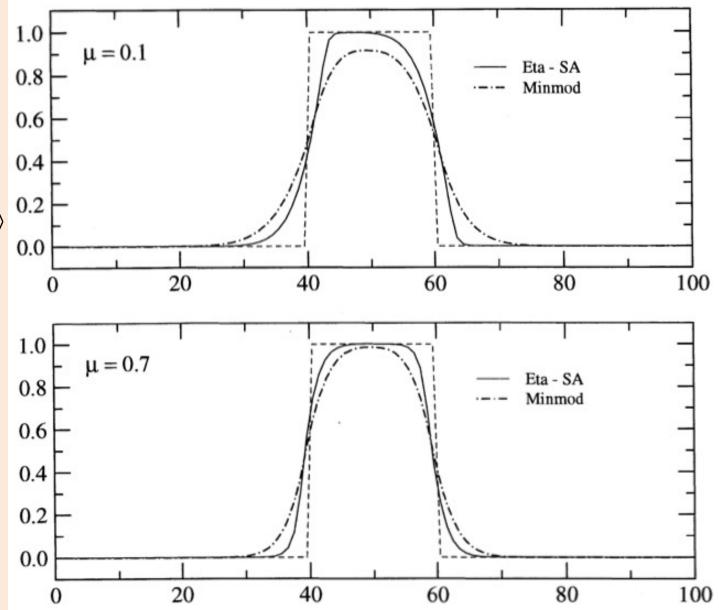


Figure 1. An example of the Eta iterative slope adjustment algorithm. The initial distribution is illustrated by the dashed line, with slopes in all five zones shown equal to zero. Slopes regulting from the first iteration are shown by the solid lines. See text for additional detail.

Minmod limiter: C = C(r), $r \equiv \frac{q_j - q_{j-1}}{q_{j+1} - q_j}$  $C(r) = \max \left< 0, \min(1, r) \right>$ defines slope to be that of the smaller, in absolute value, of the two boundary values of  $\Delta q / \Delta x$ , unless  $q_i$ is is an extremum in which case the slope is zero (Durran 1999, and also 2010, Fig. 5.16.)



After two translations of the true solution across the domain

Figure 4. Same as Fig. 2, except for the Eta slope-adjustment scheme results (SA, solid line) compared against those using the minmod slope limiter (dot-dashed line). See text for definitions of schemes.

#### Monotonizedcentered limiter:

$$C(r) = \max\left\langle 0, \min(2r, \frac{1+r}{2}, 2) \right\rangle$$

(also van Leer 1977) algebraic average of the two boundary slopes (same as using a centered scheme), unless this violates the monotonicity condition in which case they are reduced to the extent required. If however  $q_i$  is an extremum the slope is again set to zero

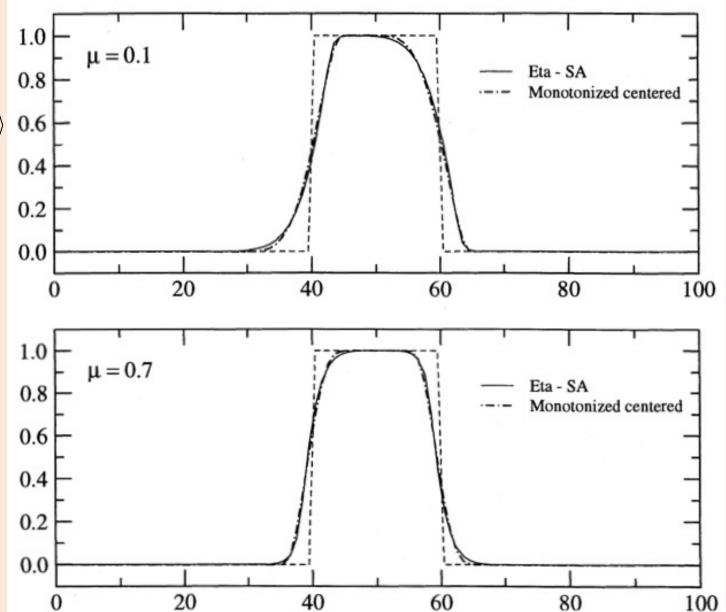


Figure 6. Same as Fig. 2, except for the Eta slope-adjustment scheme results (SA, solid line) compared against those using the monotonized centered slope limiter (dot-dashed line). See text for definitions of schemes.

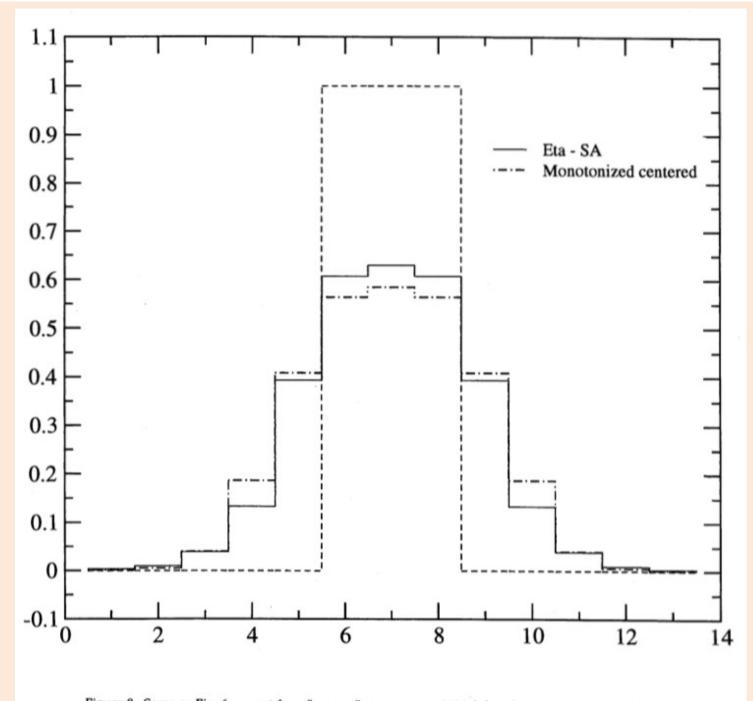


Figure 8. Same as Fig. 6, except for a "narrow" square wave initial distribution, spanning 3 zones, and a 13 zone domain.

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Takacs' 3rd order scheme (3rd order when its parameter α is a given function of μ)

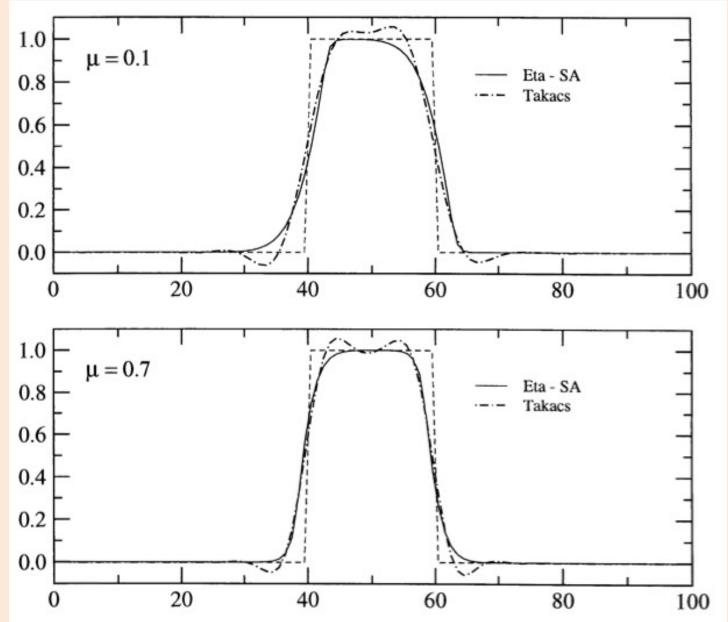
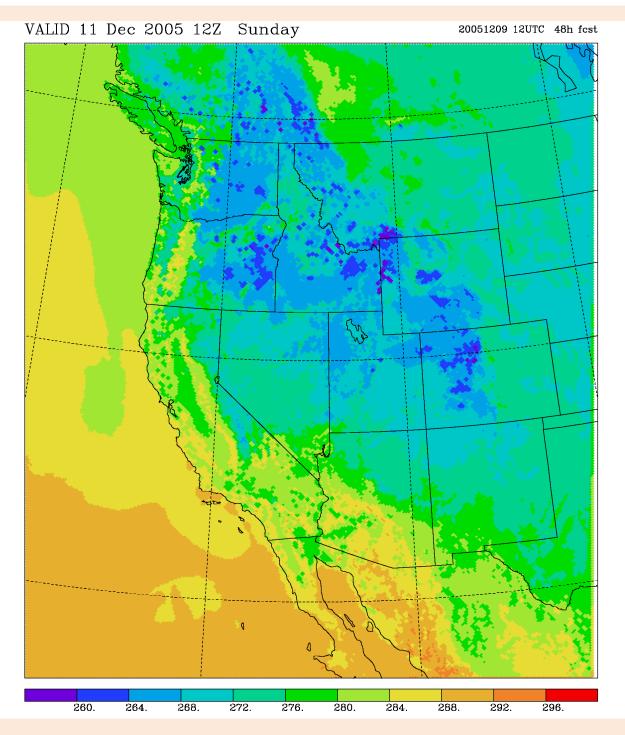


Figure 9. Same as Fig. 2, except for the Eta slope-adjustment scheme results (SA, solid line) compared against those using the Takacs (1985) third-order "minimized dissipation and dispersion errors" scheme (dot-dashed line). See text for definitions of schemes.

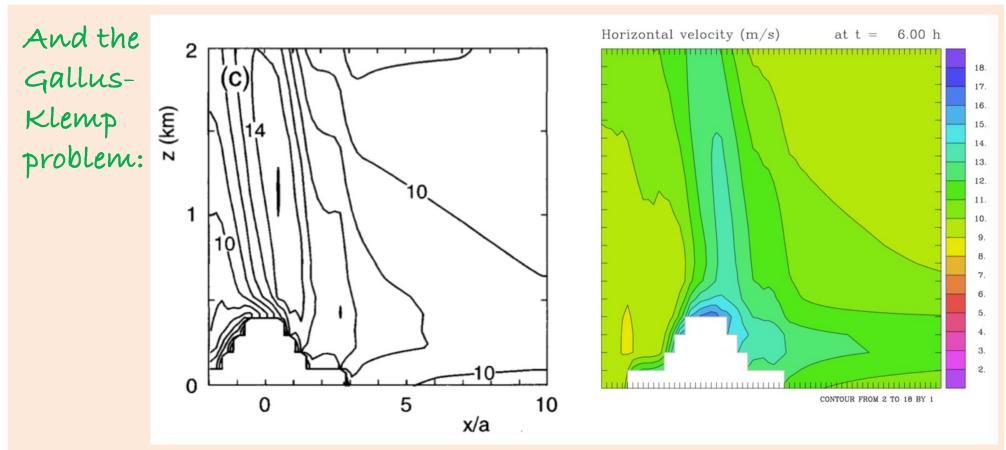
#### A still more ambitious scheme:

Rančić M., 1992: Semi-Lagrangian piecewise biparabolic scheme for two-dimensional horizontal advection of a passive scalar. *Mon. Wea. Rev.*, **120**, 1394-1406.

With finite difference scheme of slide (30) replaced by the Lagrangean slantwise advection, and the van Leer type SA scheme for vertical advections of all prognostic variables, 48-h lowest T values now



40

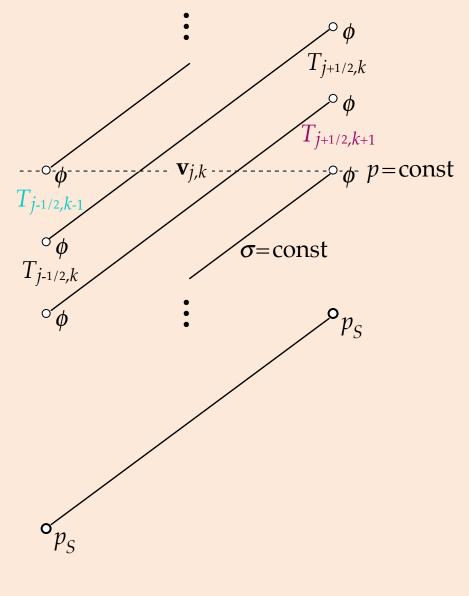


Simulation of the Gallus-Klemp experiment with the Eta code, plot (c) of Fig. 6 of Gallus and Klemp (2000), left, using the sloping steps Eta code allowing for velocities at slopes in the horizontal diffusion scheme, right. From Mesinger and Veljovic (Meteor Atmos Phys, 2017).

#### The eta vertical coordinate

Terrain-following coordinates: pressure gradient force has problems !

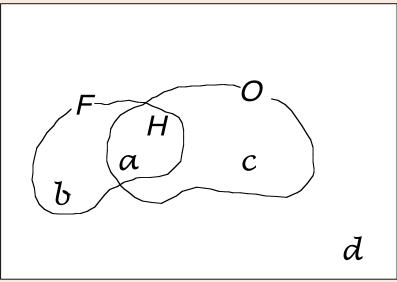
Continuous case: PGF should depend on, and only on, variables from the ground up to the p=const surface:



Accuracy of a model, ran using real data IC Issues: Atmosphere is chaotic, Results depend on data assimilation system / the IC

Impacts of both are avoided if we drive our limited area "test model" by ICs and LBCs of an ensemble of a global model Accuracy of the jet stream position ....

Forecast, Hits, and Observed (*F*, *H*, *O*) area, or number of model grid boxes:



Many verification scores. One:

$$ETS = \frac{H - E(H)}{F + O - H - E(H)}$$

<sup>4</sup> "Equitable Threat Score" or, Gilbert (1884 !) Skill Score

Bias = F/O

ETS corrected (adjusted) for bias: ETS<sub>a</sub>:

Mesinger F, 2008: Bias adjusted precipitation threat scores. *Adv. Geosci.*, **16**, 137-143 (open access).

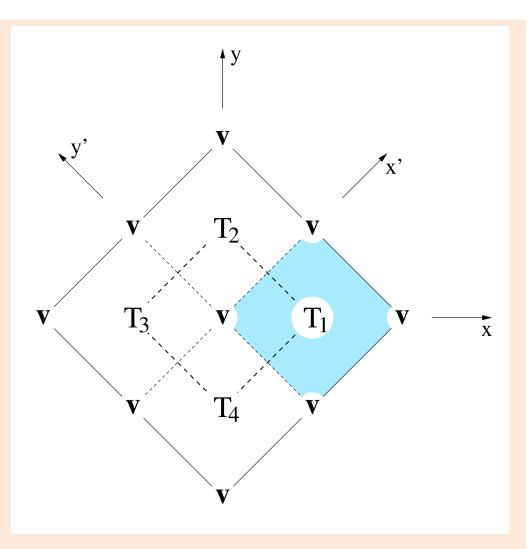
ECMWF once a week runs a 51 member ensemble forecast 32 days ahead Mesinger F, Chou SC, Gomes J, Jovic D, Bastos P, Bustamante JF, Lazic L, Lyra AA, Morelli S, Ristic I, Veljovic K (2012) An upgraded version of the Eta model. Meteorol Atmos Phys **116**, 63–79. doi:10.1007/s00703-012-0182-z

Mesinger, F, Veljovic K (2017) Eta vs. sigma: Review of past results, Gallus-Klemp test, and large-scale wind skill in ensemble experiments. Meteorol Atmos Phys, **129**, 573-593, doi:10.1007/s00703-016-0496-3

#### Horizontal treatment, 3D

Case #1: topography of box 1 is higher than those of 2, 3, and 4; "Slope 1"

Inside the central v box, topography descends from the center of T1 box down by one layer thickness, linearly, to the centers of T2, T3 and T4



Acknowledgements: Dušan Jović, Jorge Gomes, Ivan Ristić

How are grid cell values of topography obtained ?

Chop up each cell into n x n subcells;

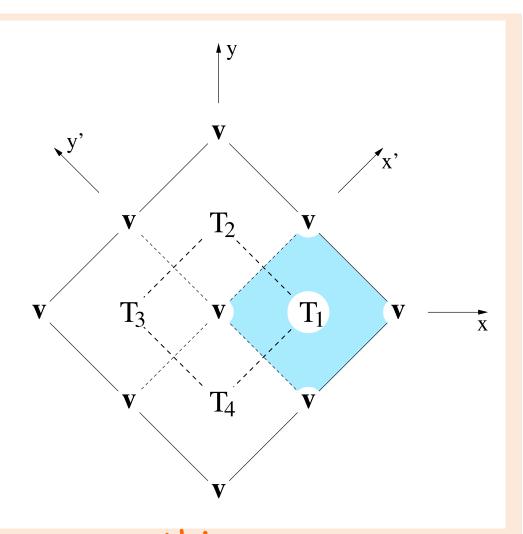
Obtain each sub-cell mean value;

Obtain mean  $h_m$  and silhouette cell value, round off to discrete interface value;

Choose one depending on Laplacian  $h_m$ ;

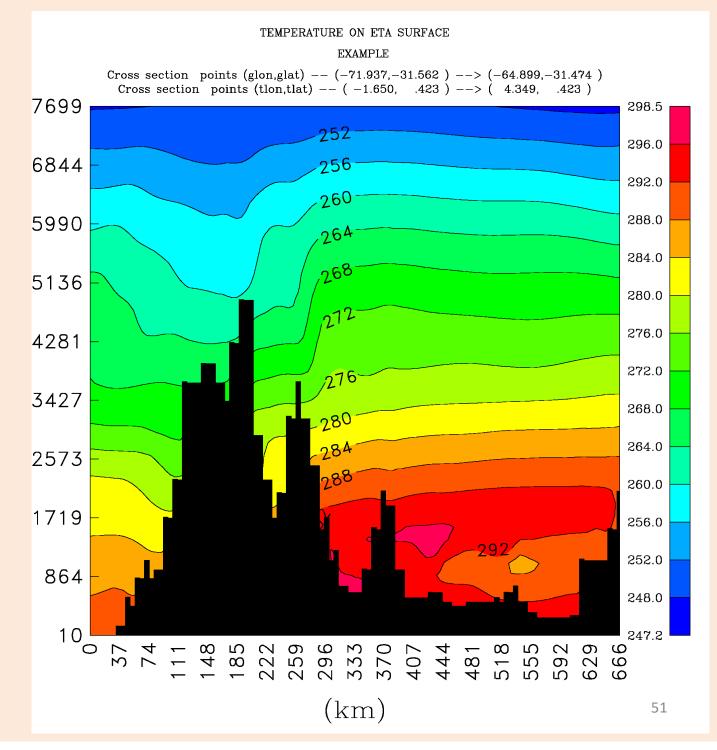
Remove basins with all corner winds blocked;

Some more common sense rules but no smoothing

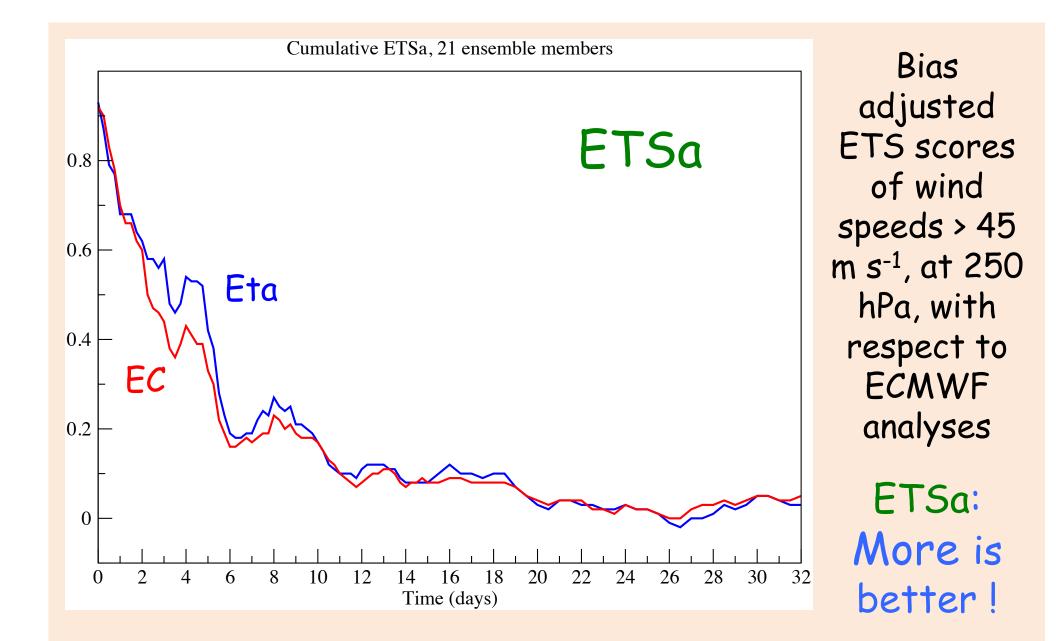


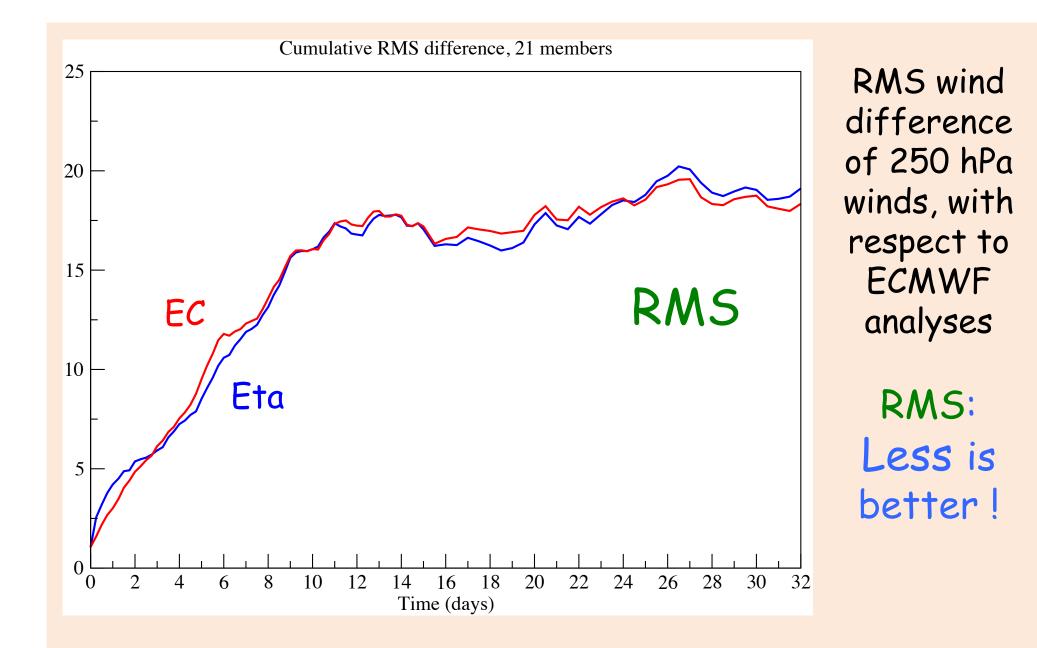
8 km horizontal resolution, W/E profile at the latitude of about the highest elevation of the Andes

NCAR graphics, no cell values smoothing



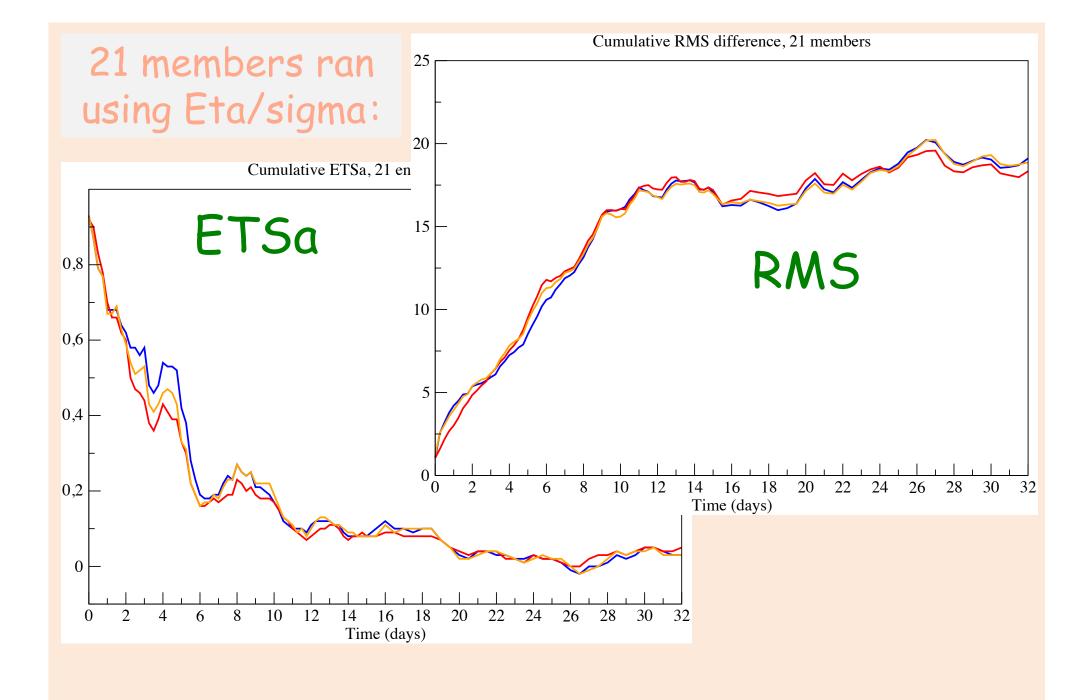
# Verification results 21 ensemble members





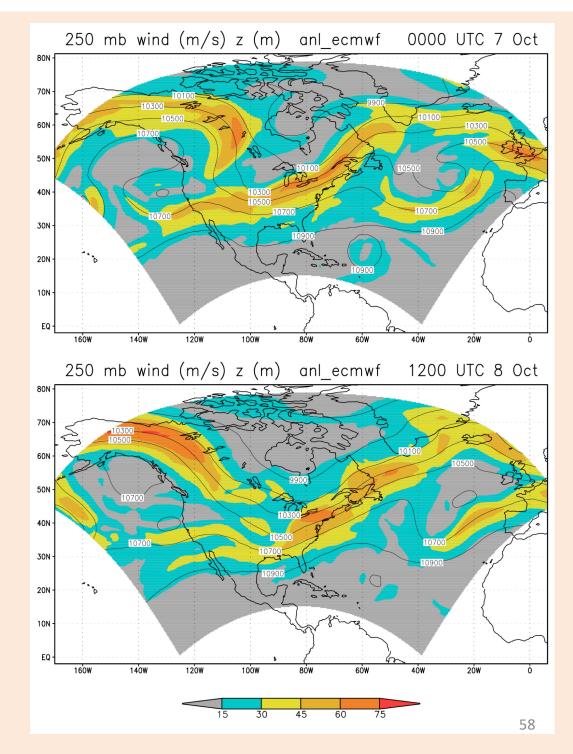
What ingredient of the Eta is responsible for the advantage in scores ?

(It is not resolution, the first 10 days resolution of two models was about the same)

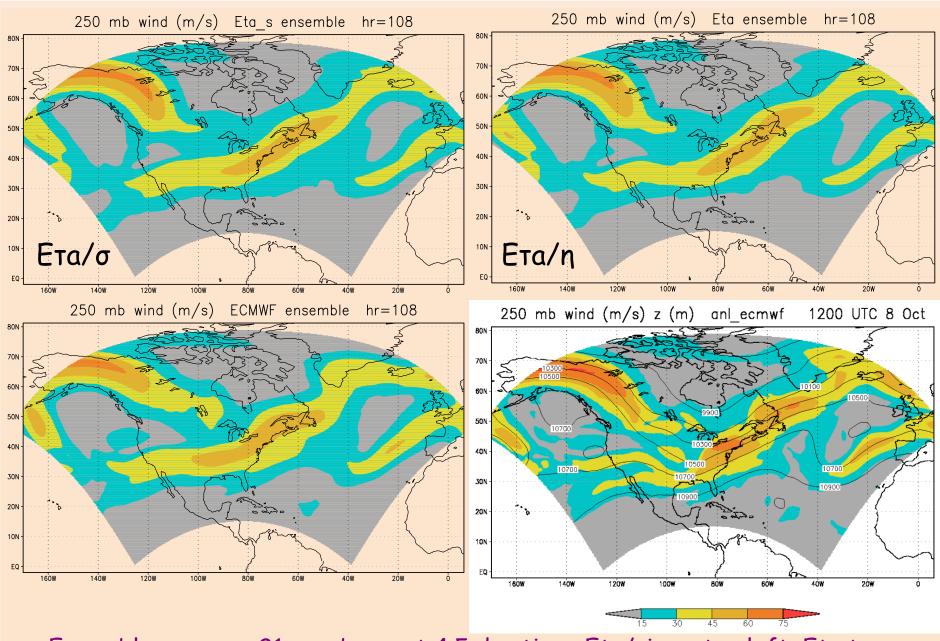


What was going on at about day 2-6 time ? What was going on at about day 2-6 time ?

The plot times correspond to day 3.0, and 4.5, respectively, of the plots of the two preceding slides

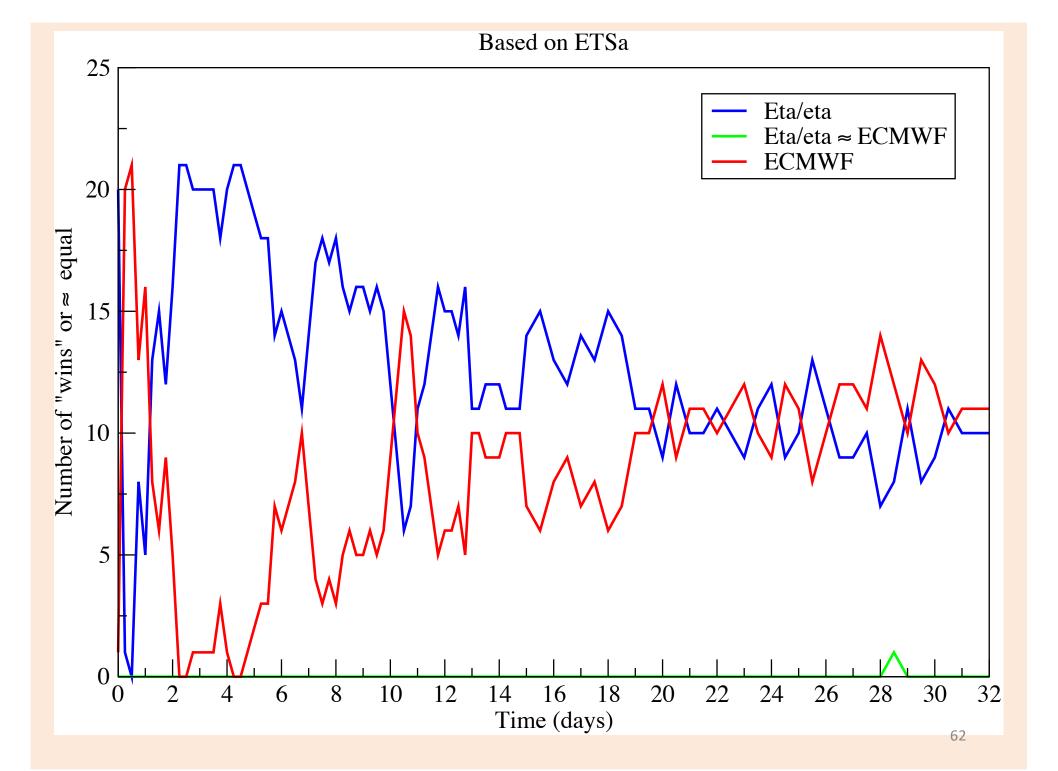


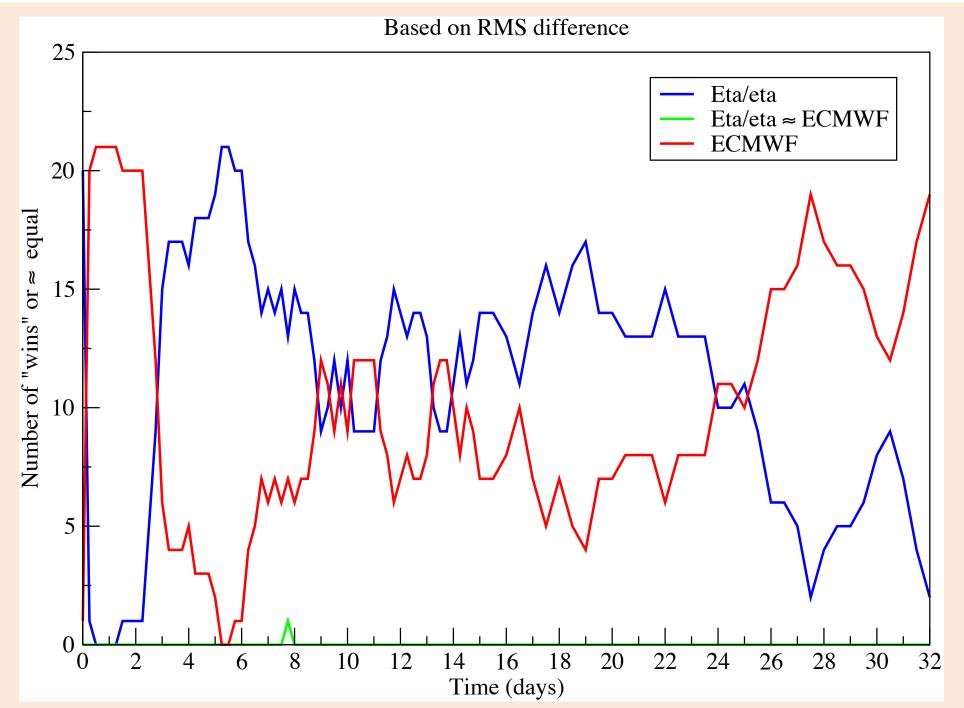
# Why was the Eta more accurate at this time?



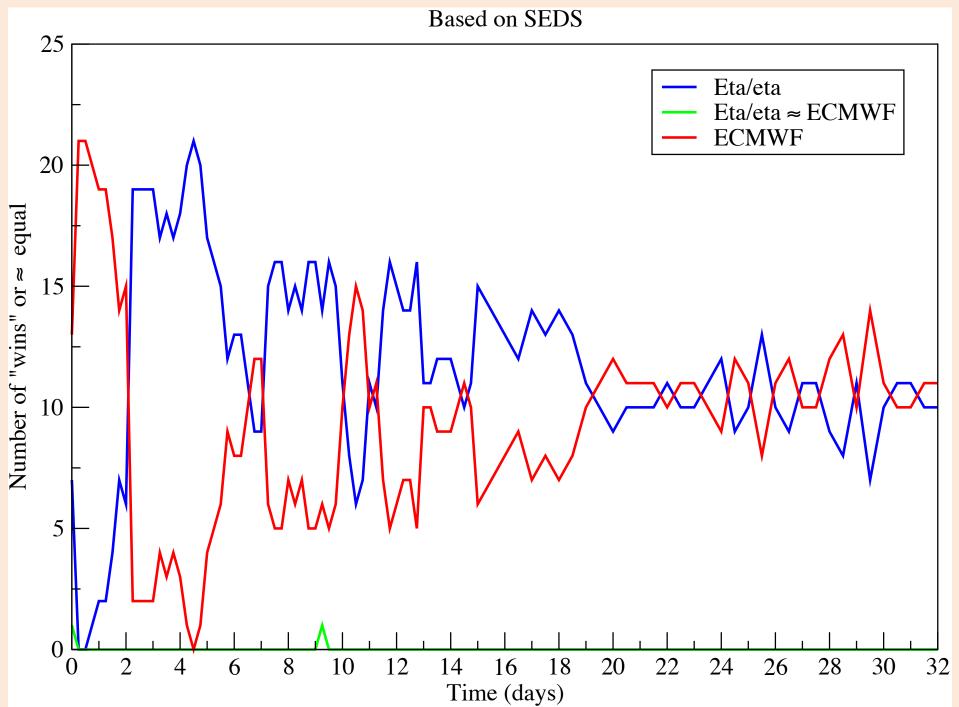
Ensemble average, 21 members, at 4.5 day time: Eta/sigma top left, Eta top right, EC driver bottom left, EC verification analysis bottom right.

### Another way of comparing ensemble model skill number of "wins"

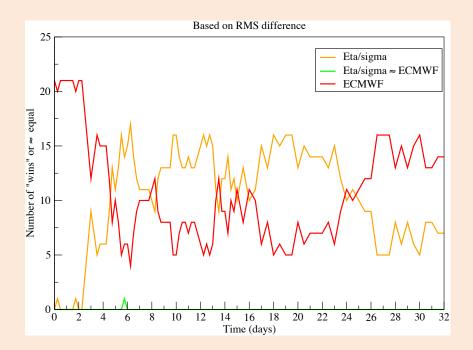


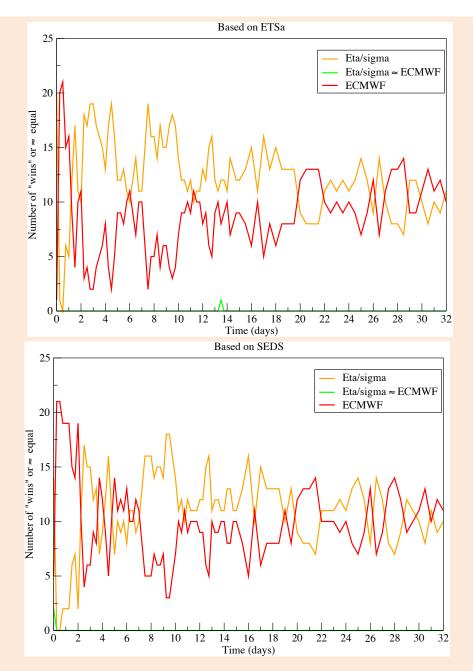


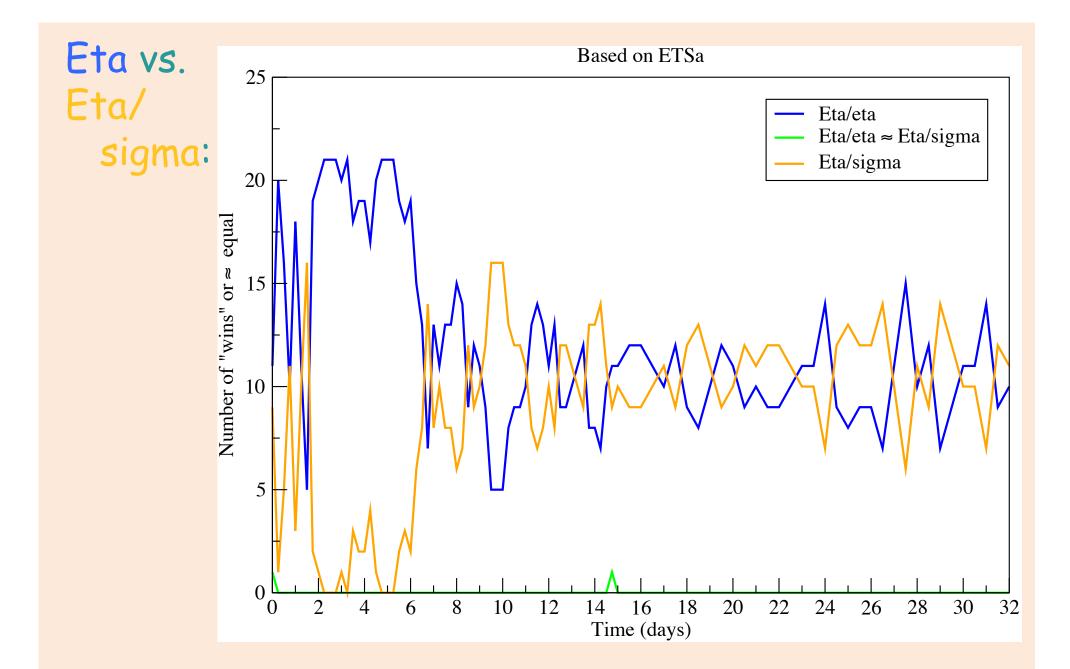
Other ways of modifying ETS (or, GSS) aimed at reducing the possibility of artificially manipulating the score, in particular by increasing bias; and its non-informative behavior for rare events (Wilks 2011, p. 313); symmetric extreme dependency score, SEDS

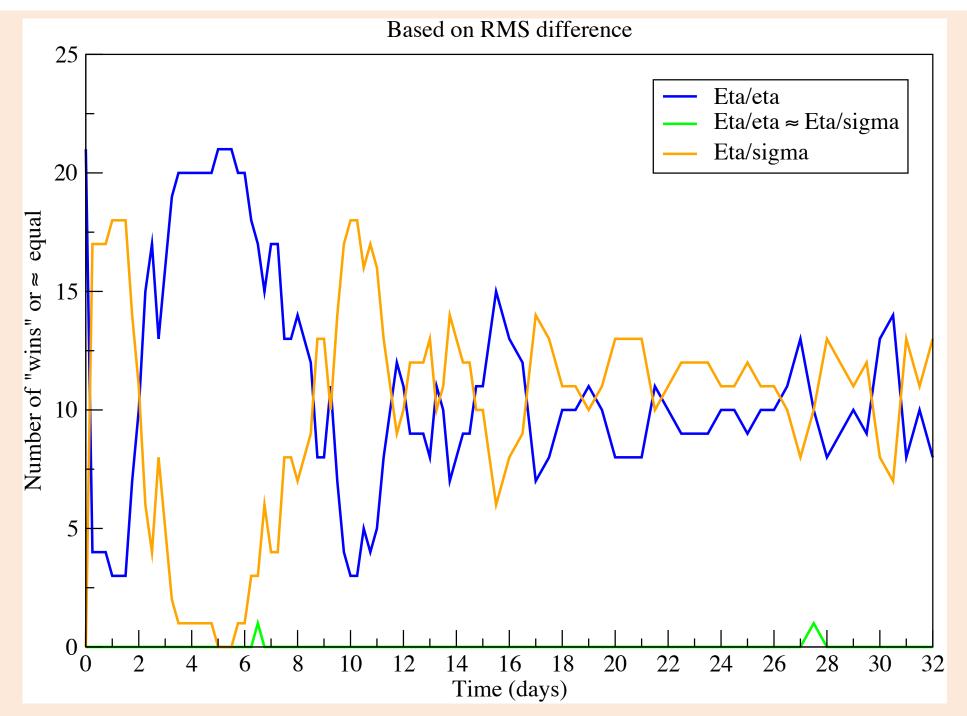


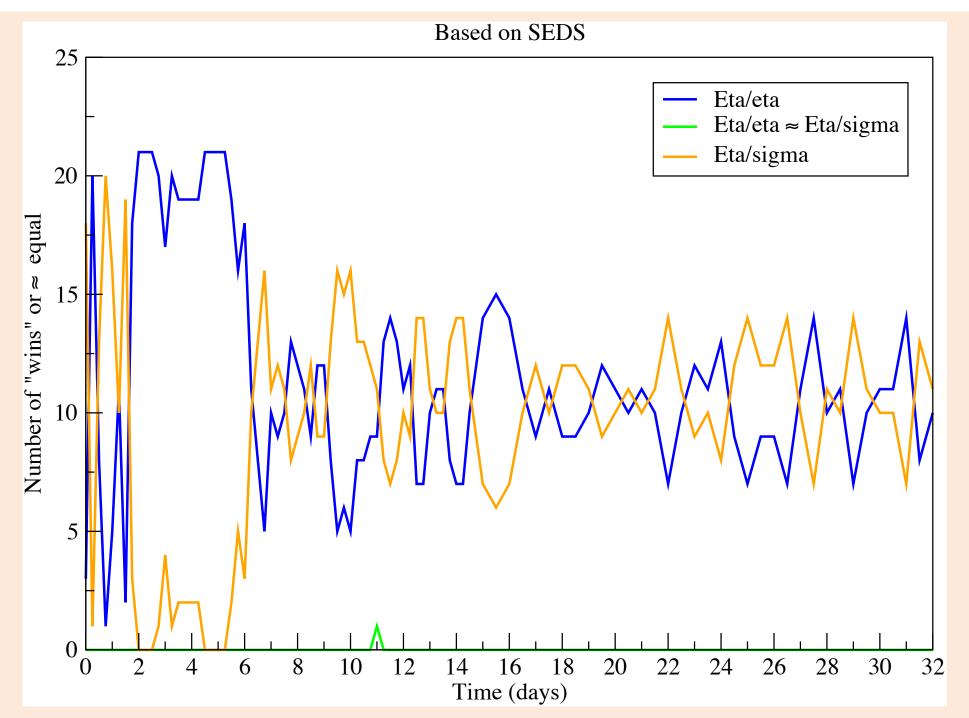
Using each of three accuracy scores, ETSa, RMS difference, and SEDS, at times ranging from 2.25 to 5.5 days, events occurred, 4, 2, 1 times, of all 21 Eta members achieving better scores than their EC driver members What happens if the Eta is switched to use sigma? Using each of three accuracy scores, ETSa, RMS difference, and SEDS, at times ranging from 2.25 to 5.5 days, events occurred, 4, 2, 1 times, of all 21 Eta members achieving better scores than their EC driver members What happens if the Eta is switched to use sigma?







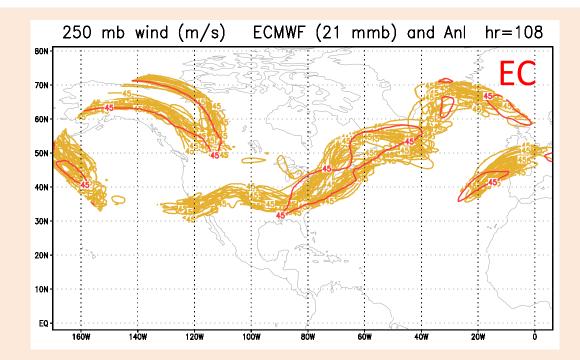




Contours of all 21 members of areas of wind speeds > 45 m/s

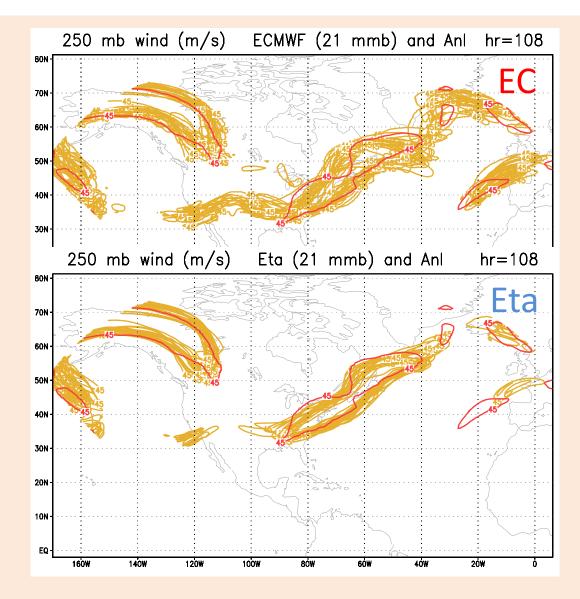
Contours of all 21 members of areas of wind speeds > 45 m/s

In red are contours of ECMWF verification analysis



Contours of all 21 members of areas of wind speeds > 45 m/s

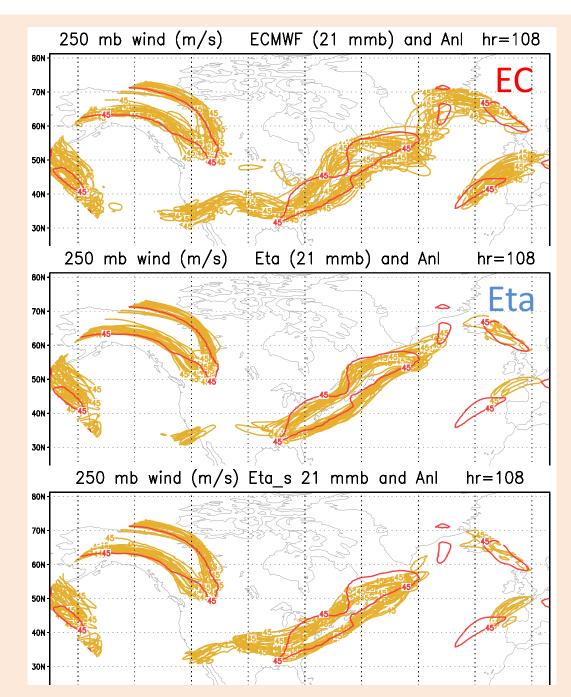
In red are contours of ECMWF verification analysis



Contours of all 21 members of areas of wind speeds > 45 m/s

In red are contours of ECMWF verification analysis

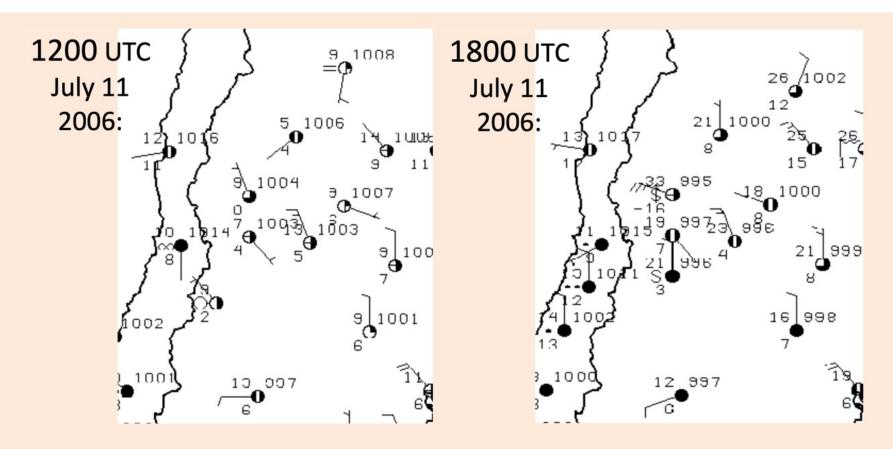
Eta/sigma :



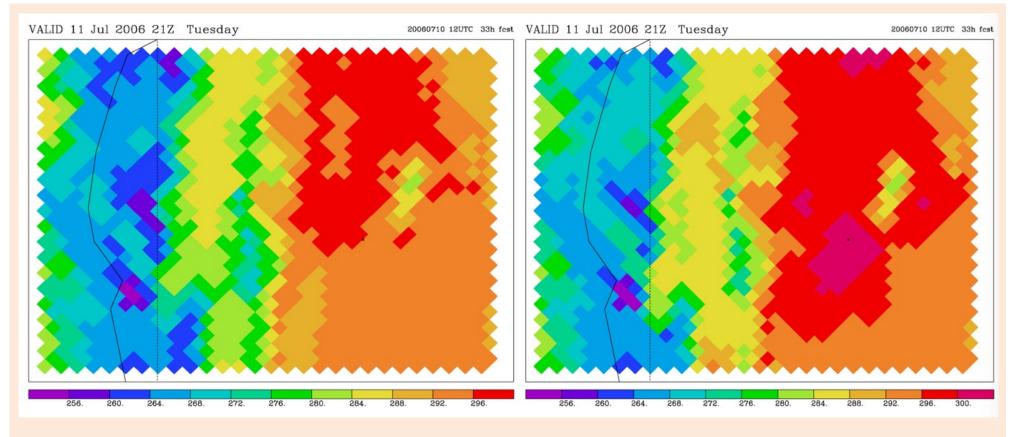
#### **Conclusion 1**

• Strong evidence that coordinate systems intersecting topography are able to perform significantly better than terrain-following systems; (in agreement with Steppeler et al. 2013)

But there must be more reasons / why is the Eta/sigma more accurate than the EC ? Look at the results of a zonda windstorm case:



Sections of surface maps illustrating a case of an intense "zonda" windstorm in the lee of the Andes. Warming from 9 to 33°C in 6 h, 24°C, is seen at the station San Juan, 630 m above sea level, close to the middle of the above sections. Valid times are displayed in the top left corner of the maps.



Forecast lowest cell temperatures at 33 h of the case discussed in Section 9 of Mesinger et al. (2012). The left-hand plot shows the result obtained using (3) for both the slantwise and the vertical advection, while the right-hand plot shows the result with these advections replaced by finite-volume schemes. The roughly vertical line on the left sides of the plots is the Chile-Argentina border, while the straight line is the 70°W meridian. The small cross to the right of the centers of plots shows the place of the San Juan station. Warming obtained in 9 h is > 20°C !

## Conclusion 2: Finite-volume vertical advection!

#### Other candidate reasons:

- Arakawa horizontal advection scheme (Janjić 1984);
- Very careful construction of model topography (MV2017), with grid cell values selected between their mean and silhouette values, depending on surrounding values, and no smoothing;
- Exact conservation of energy in space differencing in transformation between the kinetic and potential energy;

• . . . . . .

Thank you !