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ORIGINAL PAPER

Forward–backward scheme on the B/E grid modified to suppress lattice separation: the two versions, and any impact of the choice made?

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Abstract Ever since its introduction to meteorology in the early 1970s, the forward–backward scheme has proven to be a very efficient method of treating gravity waves, with an added bonus of avoiding the time computational mode of the leapfrog scheme. It has been and it is used today in a number of models. When used on a square grid other than the Arakawa C grid, modification is or modifications are available to suppress the noise-generating separation of solutions on elementary C grids. Yet, in spite of a number of papers addressing the scheme and its modification, or modifications, issues remain that have either not been addressed or have been commented upon in a misleading or even in an incorrect way. Specifically, restricting ourselves to the B/E grid does it matter and if so how which of the two equations, momentum and the continuity equation, is integrated forward? Is there just one modification suppressing the separation of solutions, or have there been proposed two modification schemes? Questions made are addressed and a number of misleading statements made are recalled and commented upon. In particular, it is demonstrated that there is no added computational cost in integrating the momentum equation forward, and it is pointed out that this would seem advantageous given the height perturbations excited in the first step following a perturbation at a single height point. Yet, 48-h numerical experiments with a full-physics model show only a barely visible difference between the forecasts done using one and the other equation forward.

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1 Introduction

It was at a meeting in May 1973 that Gadd (1974) reported on his use of a scheme for pure gravity wave terms, which he referred to as economical explicit scheme, with a reference to Ames (1969). With the scheme, the momentum equation is integrated forward, and the continuity equation backward, or the other way around, in a linear case resulting in a neutral solution within the time step twice that of the leapfrog scheme. It was shown in Mesinger (1977) that one need not be concerned with the first-order time accuracy of each of these two individual schemes, since the difference analog of the wave equation, same irrespective of which of the two equations is first integrated forward, is the simplest time-centered second-order analog of the wave equation.

All of this, obviously, is a very appealing outcome, so that the scheme has eventually gained considerable respect. Note, e.g., that in a recent analysis of the NCAR Community Climate System Model (CCSM) finite-volume dynamical core, Skamarock (2008) in two different instances points to downsides of the core's scheme by comparisons against what he refers to as the "Mesinger's forward–backward scheme". In one of those, he notes that the "modified" core's forward–backward scheme has a stability region twice as restrictive as the original and is less accurate, and in another that the forward–backward scheme cannot be used directly for a step of the core's scheme.

If the forward–backward scheme is used on a square grid other than the Arakawa C grid, a problem appears—just as with other gravity wave schemes—with the propagation of gravity wave perturbations between neighboring grid points. If, as customarily done, we analyze the problem using shallow water equations, and if the simplest centered differencing is used, a gravity wave generated by a height perturbation at a single grid point will be unable to propagate to its nearest height points (Mesinger 1973, M1 further on). As a result, a noisy pattern will be created. Lattice separation or two-grid-interval noise is the term used to refer to the problem.

Considering the Arakawa B/E grid, in M1 a method has been proposed to suppress the separation of solutions by introducing auxiliary velocity points midway between the neighboring height points, and then use acceleration contributions at these points in the calculation of the divergence term in the continuity equation. The schemes arrived at were referred to as "acceleration-modified" (AM) schemes. It was, however, pointed out that just as well "the divergence equation can be used to evaluate the local change in velocity divergence from the beginning to the middle or the end of the time step." This seems to have escaped the attention of some of the later authors, contributing to the confusion to be addressed further below.

The proposed method was applied in M1 to three time differencing schemes, these not including the forward–backward scheme because it was not known to the author at the time. The application to the forward–backward scheme, however, followed quickly thereafter (Mesinger 1974, M2 further on). Considering the linear case, it was pointed out in M2 that the solution obtained is the same "for both versions of the scheme; i.e., no matter which one of the two equations—equation of motion or the continuity equation—is first integrated forward." The derivation method of using auxiliary velocity points was used once again in M2, but it was reiterated that there is an equivalent procedure of using the divergence equation and achieving the gravity wave propagation between nearest height points via the Laplacian of the height field that appears in the continuity equation. Unfortunately, this was followed by a remark that this latter procedure "is particularly convenient for obtaining analogs of complete shallow water equation or more general primitive equations," a statement that in hindsight was unjustified as will be explained later in the text.

The gravity wave coupling modification introduced in M1 for the semi-staggered B/E grid was extended by Kar (2000) for use with the unstaggered A grid. In his summary of the approaches used to suppress the generation of the two-grid-interval noise, Kar outlines the procedure of M1 pointing out that auxiliary velocity components between neighboring height grid points were introduced, and that velocity components at these points were then used for a more accurate calculation of the divergence term in the continuity equation. Further on, Kar writes:

> "This method, by design, slightly modifies the original time-difference scheme to which it is applied, but requires *additional* computing of the Coriolis and nonlinear terms at the auxiliary velocity points, which can be prohibitively expensive as pointed out by a reviewer. On the other hand, Janjic (1974, 1979) proposed an alternative method in which, unlike Mesinger's method, *no* auxiliary velocity components are introduced and *no* additional computing of the Coriolis and advection terms is necessary." (Emphases Kar)

Kar proceeds to describe his unstaggered A grid "procedure analogous to Mesinger's method," repeatedly however stating that his resulting modified family of schemes "*unlike* Mesinger's method" does not require any additional computation of the Coriolis and advection terms at the auxiliary points, while, "As pointed out by a reviewer, Mesinger's method in its original form" does, "which can be prohibitively expensive."

It may be of interest to point out that the popularity of the lattice coupling scheme remains high as it is being used not only in the Eta but also in the NCEP WRF-NMM model; note numerous references to Janjic (1979) in Janjic (2003). In one of those (p. 278), the method is referred to as "the selective filtering technique proposed in Janjic (1979)."

In the next section, possible basis or bases for the claims of the reviewer of Kar (2000) of the existence of two alternative lattice coupling methods will be explored. This will be followed by a section in which the impact of the order of integration of the two equations involved, momentum and the continuity equation, will be looked into. In Sect. 4, the alleged need for "additional computation of the Coriolis and advection terms at the auxiliary points" will be addressed. The paper will end with a summary of conclusions arrived at.

2 Issues involved

In the analysis of the noise generation mechanism on the semi-staggered B/E grid, or for simplicity just E further on as actually used, in M1 exclusively linearized shallow water equations were considered. It is these equations that represent the simplest set within which the problem is generated, and it is of course best to address a numerical problem within the minimum set of equations and terms within which the problem appears. It was pointed out that the E grid represents a superposition of two C subgrids, and

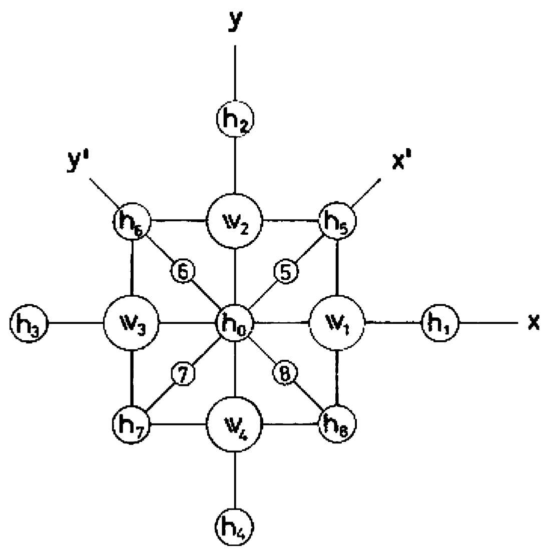

Fig. 1 E grid stencil centered on a height point, used for various explanations in the text (from Mesinger 1973, with the permission of the Swedish Geophysical Society)

that a single point perturbation propagates as a gravity wave only along the single C subgrid to which the point belongs. To couple the two subgrids, the mentioned auxiliary velocity points between neighboring height points were introduced. Schematic in Fig. 1 illustrates the approach: the auxiliary points are in the figure denoted by 5, 6, 7, and 8. The accelerations were then calculated at these points along the axes x', y' that connect the neighboring height points. This acceleration was used to arrive at velocity divergence calculated in the directions of x' and y', valid at appropriate times of the time differencing scheme at hand. As stated, the schemes arrived at were referred to as "acceleration-modified" (AM) schemes. However, the only impact of the procedure was that of a modification of the velocity divergence approximation used in the analog of the continuity equation. Thus, the term "divergence modified"—similar to "divergence corrected" used in Janjic (1974)—would have been just as appropriate, and with benefit of hindsight in fact an even better one.

It is strange to see the reviewer of the Kar paper claim that the procedure as outlined "requires *additional* computing of the Coriolis and nonlinear terms at the auxiliary velocity points" given that this is a technical statement, and that no technical support for this claim was offered. Is it even possible *to prove* that an alleged requirement to perform some additional computing cannot be avoided? I shall return to this point further below.

Let us now focus on the claim that "Janjic (1974, 1979) proposed an alternative method" and try to understand what feature or features of the cited two papers could one possibly have in mind as the basis for the reference to an *alternative* method. Note that, unfortunately, this is not spelled out, so that it is left to the readers to figure out for themselves what it is that might be thought of as a *method* different from that of M1.

The inclusion of the Janjic (1974) paper above seems puzzling given that the paper addresses specifically a problem of the 1973 method when it is applied to the leapfrog scheme. Recall that in M1 the proposed method was applied to three time differencing schemes: these were the Heun, the leapfrog, and the implicit scheme. There was a problem of weak instability when the method was applied in a straightforward way to the leapfrog scheme. Janjic (1974) managed to avoid this instability by a manipulation of the time differencing of the calculation of the velocity divergence in the modified continuity equation, and this is the content of the (Janjic 1974) paper.

Two candidate features may come to mind, one or both together, as being behind the reference to an *alternative* method. One, present in both Janjic papers, is the direct calculation of the velocity divergence in the directions of x', y' of Fig. 1, at convenient moments of the time step, as opposed to an explicit calculation of accelerations in the directions of x', y' at the auxiliary velocity points, and the subsequent calculation of the velocity divergence based on these accelerations. Clearly, the former, a direct calculation of the velocity divergence at the middle or the end of the time step, is only a more compact way of applying the same procedure, and thus arriving to the same end. No "additional terms" can appear if the *same* algebra is performed in a more explicit as opposed to a more compact way. Even so, in Sect. 4, the procedure of arriving at the "divergence modified" forward–backward scheme, which in M2 was worked out and displayed for pure gravity wave terms only, will be expanded to include the Coriolis terms, *using the auxiliary velocity points*, and it will be shown that indeed no "additional computing" is required.

An additional problem with this feature being declared as "alternative" to M1 is that this opportunity for a more compact algebra was in fact pointed out in M1 (p. 456), and once again in M2, as quoted in Sect. 1.

The other candidate feature is the difference between the application of the 1973 method to modify the forward–backward scheme, M2, and the algebra contained in Janjic (1979), in terms of the order of the integration of the momentum and the continuity equation. Namely, the equations worked out in M2 were done using the momentum equation forward version of the scheme, while those in Janjic (1979) used the continuity equation forward version. The weakness of this as the candidate feature is that it refers to one of the two cited papers only, that addressing the forward–backward scheme. This issue of the order is, however, of interest well beyond the quest of what may be meant by "alternative," and will be addressed next. Could it be that the two versions in fact are the same scheme, with merely the order of steps interchanged, or do they represent different schemes?

3 The two versions of the forward–backward scheme

We consider the linearized gravity wave terms of the shallow water equations:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}, \quad \frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y},$$

$$\frac{\partial h}{\partial t} = -H\nabla \cdot \mathbf{v},$$

with symbols here and further on having their customary meaning. The forward–backward scheme for this system, with the momentum equation integrated forward, on the E grid, is

$$\begin{aligned} u^{n+1} &= u^n - g\Delta t\delta_x h^n, \quad v^{n+1} = v^n - g\Delta t\delta_y h^n, \\ h^{n+1} &= h^n - H\Delta t\nabla_+ \cdot \mathbf{v}^{n+1}. \end{aligned} \tag{1}$$

Here, we use

$$\nabla_+ \cdot \mathbf{v} = \delta_x u + \delta_y v$$

to denote the simplest centered velocity divergence analog calculated in the directions of the x and y axes, respectively, of the E grid, depicted in Fig. 1. For brevity, we shall refer to (1) as the "MC" (momentum, continuity) scheme.

The same except for the continuity equation integrated forward, here referred to as the "CM" scheme, is

$$\begin{aligned} u^{n+1} &= u^n - g\Delta t\delta_x h^{n+1}, \quad v^{n+1} = v^n - g\Delta t\delta_y h^{n+1}, \\ h^{n+1} &= h^n - H\Delta t\nabla_+ \cdot \mathbf{v}^n. \end{aligned} \tag{2}$$

The amplification factor of (1), as well as of (2), has been analyzed at considerable length in M2 and also in Mesinger and Arakawa (1976). Specifically, it was pointed out in both of these that amplification factors of (1) and (2) are identical. This is restated in Janjic (1979).

As said above, the forward–backward scheme made its way into atmospheric modeling via a note by Gadd (1974), with a reference to Ames (1969). Interestingly, it was discovered independently much earlier and used by at least two modelers addressing the storm surge problem, Fischer (1959) and Sielecki (1968). While these authors were both impressed with the storage economy of the scheme in the sense that "no storage of old fields is necessary," neither seems to have noticed the advantage of the stability condition being twice as lenient as that of the leapfrog scheme. Forward differencing involved was a concern for Sielecki, which she however allayed by establishing energy conservation for the difference system she had. Tatsumi (1983), on the other hand, in order to avoid the first-order accuracy, developed an economical scheme of second order in each of the two equations, at a cost of complex instability problems. It was, however, demonstrated in M2 as well as in Mesinger and Arakawa (1976) that the wave equation analog of the forward–backward scheme is second-order accurate. For example, elimination of velocity components from either (1) or (2) results in

$$h^{n+2} = 2h^{n+1} - h^n + gH(\Delta t)^2\nabla_+^2 h^{n+1}, \tag{3}$$

with the Laplacian's subscript, just as in (1) and (2) and further on, denoting directions of the finite-difference operations involved. (3) is clearly the simplest space–time-centered analog of the wave equation and second-order accurate.

With the amplification factors as well as wave equation analogs of the MC and the CM scheme being identical, are these the same schemes or are they not the same schemes? Whatever one wishes to choose for an answer, what is important to note is that their *solutions* will not be the same. This is because (3) is a three time level scheme, so that the values of h^{n+2} will depend not only on the initial condition at time level n, but also on values at time level $n+1$; and these will be different when obtained using (1) or (2).

Applying the procedure of M1 to modify the MC scheme (1), we need to make a decision regarding the time centering of the acceleration contributions in the directions of x', y' of Fig. 1. It was pointed out in M2 that their time centering is advantageous to having them calculated backward in time, as it avoids a penalty in the stability condition of the scheme. Thus, one arrives at

$$\begin{aligned} u^{n+1} &= u^n - g\Delta t\delta_x h^n, \quad v^{n+1} = v^n - g\Delta t\delta_y h^n, \\ h^{n+1} &= h^n - H\Delta t\nabla_+ \cdot \mathbf{v}^{n+1} + \frac{1}{4}gH(\Delta t)^2(\nabla_\times^2 h - \nabla_+^2 h)^n, \end{aligned} \tag{4}$$

as the divergence modified version of MC. The "cross" subscript of the first Laplacian above, as well as other cross symbols further on, refer to analogs calculated using values located along the x' and y' axes of Fig. 1.

With the CM order, a somewhat less intuitive algebra needs to be used, as that in Janjic (1979), to involve accelerations in the directions of x',y'. It consists of expressing the velocity divergence analog at time level n in terms of its change from level $n-1$ to n, whereby in this change the cross Laplacian is included; followed by subsequent elimination of the velocity divergence at level $n-1$ using the momentum equation analogs of (2). One obtains

$$\begin{aligned} u^{n+1} &= u^n - g\Delta t\delta_x h^{n+1}, \quad v^{n+1} = v^n - g\Delta t\delta_y h^{n+1}, \\ h^{n+1} &= h^n - H\Delta t\nabla_+ \cdot \mathbf{v}^n + \frac{1}{4}gH(\Delta t)^2(\nabla_\times^2 h - \nabla_+^2 h)^n, \end{aligned} \tag{5}$$

the additional term of the continuity equation being of the same form as in (4). Once again, amplification factors of (4) and (5) are identical (M2; Janjic 1979).

Eliminating velocity components from (5), one obtains

$$h^{n+2} = 2h^{n+1} - h^n + gH(\Delta t)^2\left[\left(\frac{3}{4}\nabla_+^2 h + \frac{1}{4}\nabla_\times^2 h\right)^{n+1} - \frac{1}{4}(\nabla_\times^2 h - \nabla_+^2 h)^n\right], \quad (6)$$

which is identical to the wave equation analog of (4), Eq. (25) in M2.

Thus, divergence modification did not alter the property of the two versions of the scheme of having the same amplification factors as well as analogs of the wave equation. The *solutions*, for the same reason as in the case of (1) and (2), will not be the same. Which one seems advantageous? In the Eta model, (5) is used, for historical reasons. However, inspection of what happens after a physics step might raise concerns. In the Eta, following a physics step, two consecutive adjustment steps are done. Imagine a physics impact, for example heating, at a single grid point; a shallow water equivalent of this event is one of water being poured at a height point. In the first adjustment step, using (5), the continuity equation response to this perturbation will result only from the modification term, given that velocity components have not yet been perturbed. In response, heights of the four nearest height points will increase, but those of the four next nearest points will decrease! Velocity components will be perturbed in the subsequent momentum equation part of the first step. Thus, prior to getting into other code routines, in two adjustment steps the height perturbation will be acted upon by the velocity divergence term of (5) only once.

It may be of interest to consider the actual values of changes in the neighboring height points following such a height perturbation at a single point. Suppose that at the initial time, say at point h_0 of Fig. 1, a perturbation Δh is imposed, with initial values of all variables at other points being constant. With the MC order, to inspect the resulting changes in the neighboring height points, we insert velocities at level $n+1$ into the continuity equation of (4), obtaining

$$h^{n+1} = h^n - H\Delta t\nabla_+\cdot\mathbf{v}^n + \frac{1}{4}gH(\Delta t)^2(\nabla_\times^2 h + 3\nabla_+^2 h)^n.$$

Thus, as a result of the perturbation Δh, changes at the nearest and at the second nearest h points at the time level $n+1$ will amount to

$$gH\left(\frac{\Delta t}{2d}\right)^2\Delta h, \quad \text{and} \quad gH\left(\frac{\Delta t}{2d}\right)^2\frac{3}{2}\Delta h,$$

respectively.

On the other hand, with the CM order, using the continuity equation of (5), for the changes at the same two sets of points we have

$$gH\left(\frac{\Delta t}{2d}\right)^2\Delta h, \quad \text{and} \quad -gH\left(\frac{\Delta t}{2d}\right)^2\frac{1}{2}\Delta h.$$

The preceding, and in particular the initial perturbation at the second nearest points of the wrong sign with the CM version, would seem to suggest that the MC version should be advantageous. Even so, in two real data experiments performed using Eta model versions with full physics, at 48 h differences barely noticeable were obtained. One of these was on a case of very heavy rains over the northern California topography. Another was also on a case of heavy rains, but over Scandinavia. In both of these, on sea level pressure and precipitation maps inspected, differences obtained using the MC and CM versions were extremely small to the extent of being difficult to notice. For illustration, the precipitation maps for the Scandinavia case are shown in Fig. 2. The figure displays 48-h accumulated precipitation obtained in forecasts initialized at 0000 UTC 28 February 2006, using the MC (upper panel) and the CM version (lower panel), respectively. For more detail on the model setup, see Popovic (2006). Differences in sea level pressure maps, and in the California case in both precipitation and sea level pressure, were of similar, hardly visible kind.

Could it be that more of an impact would be obtained in still longer integrations? Note that in the global Eta model (Zhang and Rancic 2007), the MC version is used. Further experiments and increased understanding seem desirable.

4 Coriolis force with the momentum equation forward

To extend the preceding MC version considerations to include the Coriolis force, using auxiliary velocity points we write

$$u^{n+1} = u^n - g\Delta t\delta_x h^n + f\Delta t\frac{1}{2}(v^n + v^{n+1}), \quad (7.1)$$

$$v^{n+1} = v^n - g\Delta t\delta_y h^n - f\Delta t\frac{1}{2}(u^n + u^{n+1}), \quad (7.2)$$

$$h^{n+1} = h^n - H\Delta t\left(\frac{1}{2}\nabla_+\cdot\mathbf{v}^{n+1} + \frac{1}{2}\nabla_\times\cdot\mathbf{v}^{(n+1)*}\right), \quad (7.3)$$

where the analog to be used for the cross divergence in (7.3), $\nabla_\times\cdot\mathbf{v}^{(n+1)*}$, has yet to be determined. The convenience of using the trapezoidal implicit scheme for the Coriolis force terms along with the forward–backward scheme for the gravity wave terms was noted very early by Fischer (1965).

Recall that the continuous system we are now approximating can be used to obtain the divergence equation:

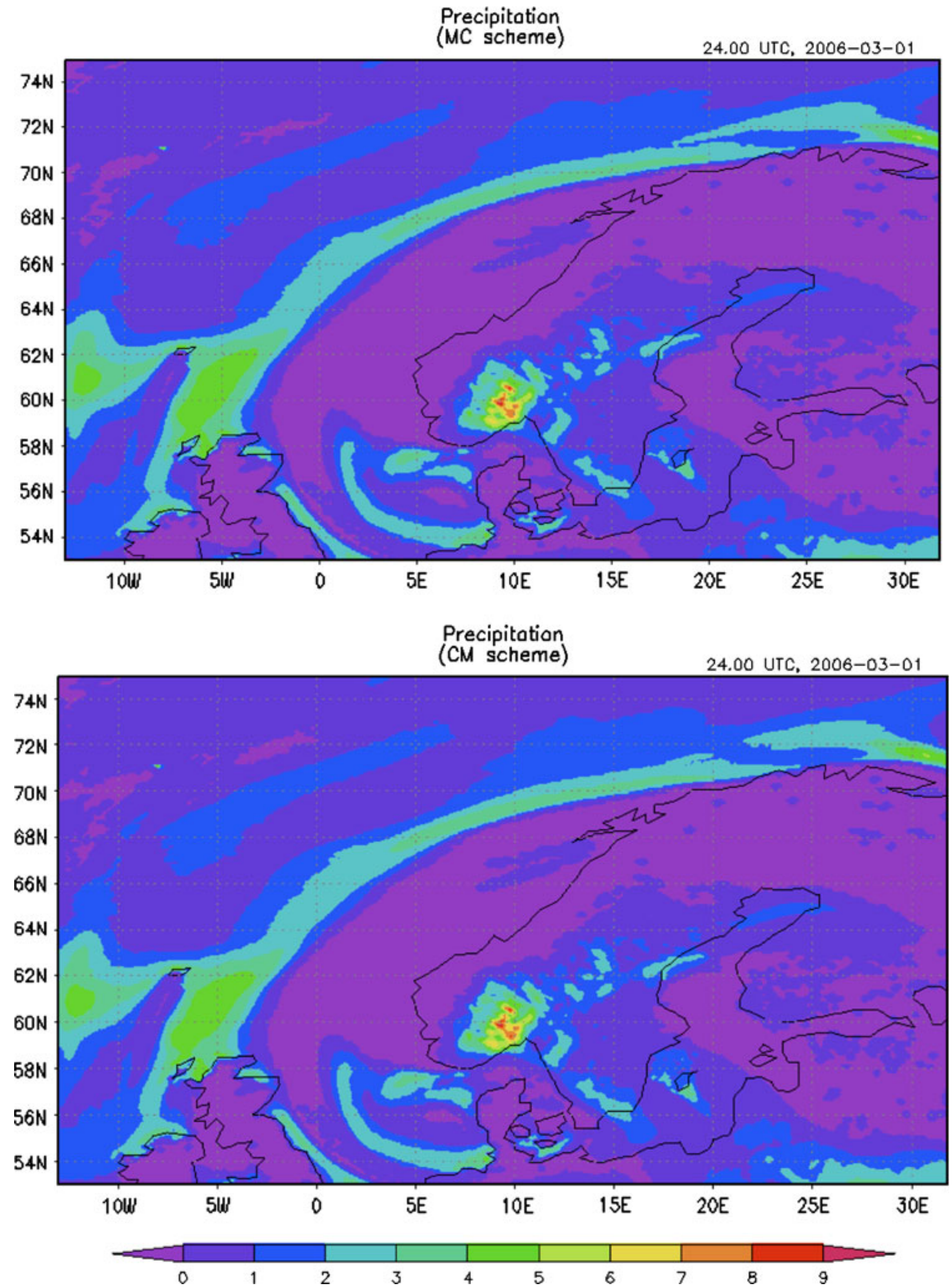

Fig. 2 48-h accumulated precipitation, mm, in Eta model forecasts done using the momentum–continuity order within the model's forward–backward scheme for the gravity-inertia terms (*upper*), and using the continuity–momentum order (*lower*). See text for additional detail

$$\frac{\partial}{\partial t}\nabla \cdot \mathbf{v} = -g\nabla^2 h + f\zeta.$$

We want to use it to arrive at the definition of the cross divergence term in (7.3) involving the auxiliary velocity components at points 5, 6, 7, and 8 of Fig. 1. These velocity components, u' and v' in the directions of x' and y', respectively, are taken to be equal to averages of the velocity components in the same directions at two nearest regular velocity points where velocities are carried, denoted as $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ in the figure. For the Coriolis term, emulating (7.1) and (7.2), we use the time average of the analog of cross vorticity:

$$\zeta_\times = \delta_{x'} v' - \delta_{y'} u'.$$

Thus, we have

$$\frac{1}{2}\nabla_\times \cdot \mathbf{v}^{(n+1)*} = \frac{1}{2}\nabla_\times \cdot \mathbf{v}^n - \frac{1}{2} g\Delta t \nabla_\times^2 h^n + \frac{1}{4} f\Delta t(\zeta_\times^n + \zeta_\times^{n+1}). \tag{8}$$

Straightforward algebra, however, shows that

$$\nabla_\times \cdot \mathbf{v} = \nabla_+ \cdot \mathbf{v} \tag{9}$$

so that, using (7.1) and (7.2), the first term on the right hand side of (8) can be written as

$$\frac{1}{2}\nabla_\times \cdot \mathbf{v}^n = \frac{1}{2}\nabla_+ \cdot \mathbf{v}^n = \frac{1}{2}\nabla_+ \cdot \mathbf{v}^{n+1} + \frac{1}{2}g\Delta t\nabla_+^2 h^n - \frac{1}{4}f\Delta t(\zeta_+^n + \zeta_+^{n+1}), \quad (10)$$

with ζ_+ being the standard centered second-order analog of vorticity:

$$\zeta_+ = \delta_x v - \delta_y u.$$

Just as with (9) though, straightforward algebra shows that

$$\zeta_\times = \zeta_+. \quad (11)$$

Thus, using (8) and (10), (7.3) leads to

$$h^{n+1} = h^n - H\Delta t\nabla_+ \cdot \mathbf{v}^{n+1} + \frac{1}{2}gH(\Delta t)^2(\nabla_\times^2 h - \nabla_+^2 h)^n.$$

Just as in M2 and unrelated to the Coriolis terms, the modification term is obtained with a weight twice that required for no penalty in the stability condition, as a result of using the forward as opposed to centered analog of the momentum equation. For no penalty, as in M2 and (4), only half of the weight can be taken, equivalent to time centering, or to averaging this and the third equation of (1). Thus, we obtain:

$$u^{n+1} = u^n - g\Delta t\delta_x h^n + f\Delta t\frac{1}{2}(v^n + v^{n+1}),$$
$$v^{n+1} = v^n - g\Delta t\delta_y h^n - f\Delta t\frac{1}{2}(u^n + u^{n+1}),$$
$$h^{n+1} = h^n - H\Delta t\nabla_+ \cdot \mathbf{v}^{n+1} + \frac{1}{4}gH(\Delta t)^2(\nabla_\times^2 h - \nabla_+^2 h)^n, \quad (12)$$

as the divergence modified gravity-inertia wave analog looked for. Specifically, no additional terms have appeared in (12) because of the procedure based on the use of the auxiliary velocity points, 5, 6, 7, and 8 in Fig. 1.

It seems not necessary and in fact pointless to go into the analogous algebra regarding the advection terms, since the forward–backward scheme is intrinsically a split-type scheme requiring a separate time differencing for non-gravity-inertia terms. Accordingly, in the Eta, and also in the WRF-NMM, advection is calculated in a split mode, following the adjustment steps or step.

5 Summary

Several unsettled issues of the forward–backward scheme modified to suppress lattice separation have been looked at. Of these, an intriguing one is that of the possible impact of the order of integration of the momentum and the continuity equation. One should recall that nowhere in the papers of Mesinger (1974) and Janjic (1979) has it been claimed that the two versions of the scheme, in their original or modified forms, are identical; but the claim that they are different was also not made. It is pointed out that not only the amplification factors of the two versions of the original and the modified schemes are the same, but that even their difference wave equation analogs are the same. However, even so, their *solutions*, starting from a given initial condition, will not be the same. This is because of the different result at the first time step. In a full-blown atmospheric model, with forcing primarily of the height (temperature) field, it is pointed out that integrating the momentum equation forward seems advantageous, because of not producing changes of the wrong sign at grid points that are second nearest to a perturbation point. However, two real data experiments performed failed to identify a noticeable impact. Thus, the impact, while possibly noticeable in idealized cases or in some real data cases and/or longer integrations, is not likely to be significant.

Efforts were made to identify reasons to refer to Janjic (1974, 1979) as a source for a method *alternative* to that of Mesinger (1973), as done by an anonymous reviewer of Kar (2000). These efforts failed since no features could be found to serve that purpose. Specifically, it was demonstrated as incorrect that using the method of Mesinger (1973) based on auxiliary velocity points "requires additional computing of the Coriolis and nonlinear terms at the auxiliary velocity points." This should have been expected, since using auxiliary velocity points is merely a more explicit version of the design of "divergence modified" schemes, whereby the two-grid-interval noise is suppressed, than that of using the divergence equation for the same purpose directly.

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