# Mass Flux Scheme

- The single component of the cloud is not treated individually but as bulk effects produced by an ensemble of clouds.

- The large scale area average mass flux, $\overline{M}$ , is assumed to contain an environment part, represented as $M_e$ , and a cloudy part, $M_c$ , which represents the contribution from all clouds.

$$\overline{M} = M_c + M_e$$

- The clouds occupy a fractional area $\sigma$ , and the environment $1-\sigma$

- Vertical mass fluxes can be rewritten as

$$\overline{\rho}\,\overline{w} = \sigma \overline{\rho} w_c + (1-\sigma)\overline{\rho} w_e$$

# Mass Flux Scheme

•The observed $\overline{w}$ is generally small, which implies that the strong ascent within the cloud is compensated by the descent between clouds.

From the assumption that $\sigma << 1$ , and that $\overline{w} \approx 0$ , we have then

$$\overline{\rho}\,\overline{s'w'} \approx \sigma \overline{\rho} w_c (s_c - s_e) = M_c (s_c - s_e)$$

$s$ : dry static energy

# Mass Flux Scheme

A cloud model is necessary to give values of $M_c$ and $s_c$

Based on the mass conservation and assuming steady state

$$\frac{\partial \rho_d}{\partial t} + \nabla.(\rho_d V) = 0 \qquad \therefore \qquad \frac{\partial \rho w_c}{\partial z} = E - D$$

$E$ and $D$ are the entrainment and detrainment rates

$$\frac{\partial M_c}{\partial z} = E - D$$

Extension to other typical cloud conservative properties:

$$\frac{\partial (M_c s_c)}{\partial z} = E\bar{s} - Ds_c$$

In a moist atmosphere:

$$\frac{\partial (M_c s_c)}{\partial z} = E\bar{s} - Ds_c + L\rho c$$

# Mass Flux Scheme

The conservation of water substance can be split into vapour, $q$, and liquid, $l$, phases.

$$\frac{\partial(M_c q_c)}{\partial z} = E\bar{q} - Dq_c - \rho c$$

$$\frac{\partial(M_c l)}{\partial z} = -Dl + \rho c - \rho k l$$

$k$ is the rate of conversion of liquid water into precipitation.

By writing the convective eddy transports in the flux form, the energy conservation in the column is assured.

# Mass Flux Scheme

The contribution from convective activity to the large scale heat and moisture are thus,

$$\left(\frac{\partial\bar{\theta}}{\partial t}\right)_{cu} = -\frac{1}{\bar{\rho}}\frac{\partial\left[M_c(\theta_c-\bar{\theta})\right]}{\partial z} + \frac{L}{c_p\bar{\pi}}(c-e)$$

$$\left(\frac{\partial q}{\partial t}\right)_{cu} = -\frac{1}{\bar{\rho}}\frac{\partial\left[M_c(q_c-\bar{q})\right]}{\partial z} - (c-e)$$

Observations show that cloud mass flux is larger than the vertical mass flux forced by large scale convergence.

The representation of cloud mass flux from large scale convergence is not enough to reproduce the warming in cloud free area. There is need for an explicit representation of mass transports, or of other quantities, within the cloud.

# KAIN-FRITSCH CONVECTION PARAMETERIZATION

- Based on Fritsch-Chappell Scheme
- Based on Mesoscale Convective Systems
- Mass Flux Type
- Cloud Model to estimate the convective mass fluxes

# Trigger Function:

1. $$T_{LCL} + \Delta T - T_{ENV} \begin{cases} >0 \Rightarrow unstable \longrightarrow \text{Deep convection} \\ \le 0 \Rightarrow stable \end{cases}$$

$$\Delta T = k(w_g - c_z)^{\frac{1}{3}}$$

$w_g$ : background vertical motion at LCL

$c_z = 2$, $z_{LCL} > 2000$

$c_z = 2*(z_{LCL}/2000)$, $z_{LCL} < 2000$

Parcel is given an extra temperature perturbation

2. $$w_{po} = 1 + 1.1\left(\frac{\Delta T}{T} Hpbl\right)^{1/2}$$

**Dilute** parcel ascent

$w_p > 0$ Within cloud depth (3-4km)

3. $$D_{\min} = \begin{cases} 4000 & , \quad T_{LCL} > 20^O C \\ 2000 & , \quad T_{LCL} < 0^O C \\ 2000 + 100 \times T_{LCL} & , \quad 0 \le T_{LCL} \le 20^O C \end{cases}$$

Minimum cloud depth

## Updraft:

$$\frac{\partial M_u}{\partial z} = \varepsilon_u - \delta_u$$

$$\frac{\partial M_u \theta_u}{\partial z} = \varepsilon_u \theta - \delta_u \theta_u + L(c-e)$$

$$\frac{\partial M_u q_u}{\partial z} = \varepsilon_u q - \delta_u q_u - L(c-e) - P$$

$$\frac{\partial M_u l}{\partial z} = -\delta_u l - P + L(c-e)$$

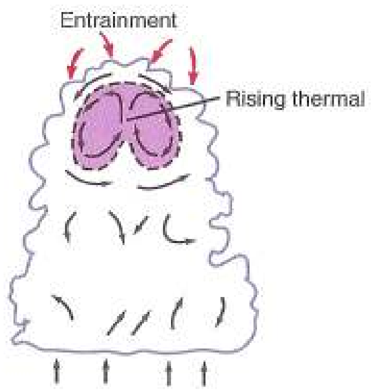



Fig. 6.12 Schematic of entrainment of ambient air into a small cumulus cloud. The thermal (shaded violet region) has ascended from cloud base. [Adapted from *J. Atmos. Sci.* **45**, 3957 (1988).]

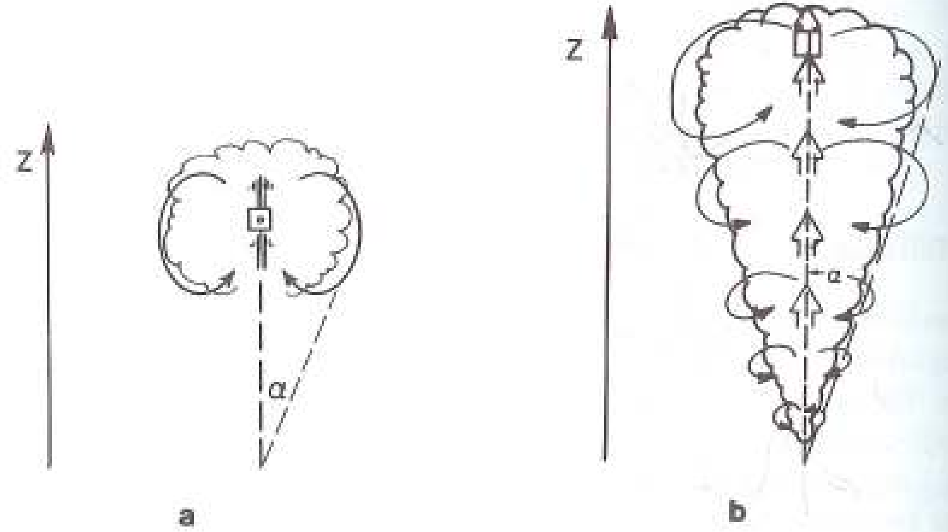
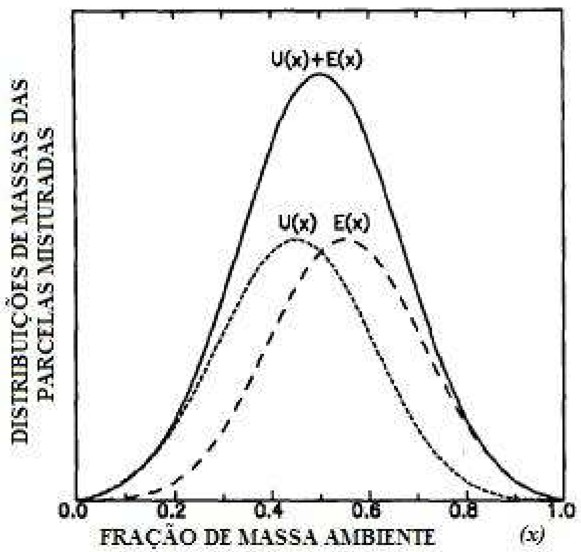



**Fig. 8.33.** (a) Schematic view of the "bubble" or "thermal" model of lateral entrainment in cumuli. (b) Schematic view of the "steady-state jet" model of lateral entrainment in cumuli.

•**Lateral entrainment** : injection of environmental air into the cloud. Dilution from cloud top downwards.

•**Detrainment:** cloud water lost to the environment. Cloud droplets evaporate in the unsaturated environment, cloud environment is cooled and bouyancy is increased.




increase ε in high buoyancy and/or moist environment

increase δ in low buoyancy and/or dry environment

E(x): environmental mass distribution
U(x): updraft mass distribution
f(x): gaussian mass distribuition

Entrainment rate $$M_{ee} = \delta M_t \int_0^{x_C} x f(x) dx$$

Detrainment rate $$M_{ud} = \delta M_t \int_{x_C}^{1} (1-x) f(x) dx$$

# Updraft:

Variable cloud radius, *R*: control of entrainment rate

$$\delta M_e = M_{uB} \frac{(-0.03 * \delta p)}{R}$$

$\delta M_e$ : maximum possible entrainment rate

*R* dependent on large scale forcing through grid-scale vertical motion

$$R = \begin{bmatrix} 1000, W_{KL} < 0 \\ 2000, W_{KL} > 10 \\ 1000 + \frac{W_{KL}}{10}, 0 \le W_{KL} \le 10 \end{bmatrix}$$

$W_{KL}$ is proportional to $w_g$

Weaker dilution when low-level forcing is stronger

## Downdraft:

- LFS: Level of Free Sink (DSL : downdraft source level) occurs about 150-200 hPa above cloud base

- downdraft source is environmental air only

- downdraft ends when it becomes warmer than the environment or reaches the surface.

$$\frac{\partial M_d}{\partial z} = \varepsilon_d - \delta_d$$

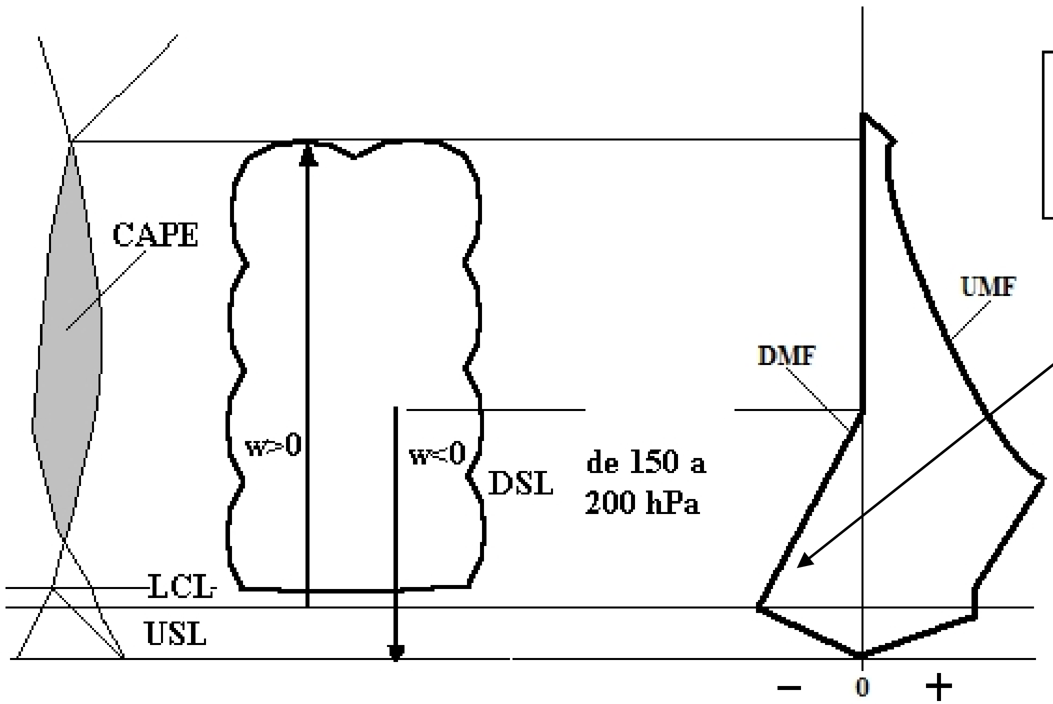
$M_d$ at DSL to start derivation

$$\frac{\partial M_d \theta_d}{\partial z} = \varepsilon_d \theta - \delta_d \theta_d - Le$$

$$M_{dDSL} = M_{uB} * (2 \times (1 - \overline{RH}_{DSL})$$

$$\frac{\partial M_d q_d}{\partial z} = \varepsilon_d q - \delta_d q_d + Le$$

# Downdraft:




$$\frac{DMF_{USL}}{UMF_{USL}} = 2 \times \left(1 - \overline{RH}\right)$$

# Shallow convection:

- Trigger: same as deep convection but cloud depth smaller than the minimum cloud depth, $D_{min}$
- No precipitation produced
- R is constant, entrainment rate is constant
- $M_{uB} \propto$ max TKE in subcloud layer

$$M_{u0} = \begin{cases} \left(\frac{TKE_{MAX}}{k_0}\right) \times \left(\frac{m_{USL}}{\tau_C}\right) & , \quad TKE_{MAX} < 10 \\ \left(\frac{10}{k_0}\right) \times \left(\frac{m_{USL}}{\tau_C}\right) & , \quad TKE_{MAX} \geq 10 \end{cases}$$

$\tau_C$ é o período de tempo convectivo, variando de 1800 a 3600 [s];
$m_{USL}$ é quantidade de massa no USL [kg];
ko valor de referência [$m^2$ $s^{-2}$].

## CLOSURE:

- **0.9*CAPE in the column is removed within $t_c$**
- **CAPE is calculated from <u>dilute</u> parcel ascent > smaller $M_{uB}$**
- **CAPE is removed by lowering $\theta_e$ in the USL and warming environment aloft**

## Tendencies:

$$\left.\frac{\partial \theta}{\partial t}\right|_{con} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left[\left(M_u + M_d\right)\theta + \left(\varepsilon_u + \varepsilon_d\right)\theta - \left(\delta_u + \delta_d\right)\theta_u\right]$$

$$\left.\frac{\partial q}{\partial t}\right|_{con} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left[\left(M_u + M_d\right)q + \left(\varepsilon_u + \varepsilon_d\right)q - \left(\delta_u + \delta_d\right)q_u\right]$$

Flux form: Conservation of moisture and energy

# Convective momentum fluxes (KF,93)

## Zonal momentum equation

$$\left.\frac{\Delta\bar{u}}{\Delta t}\right|_{CONV} = \frac{1}{\Delta p}\left[\left(\omega_{u2}+\omega_{d2}\right)\bar{u}_2 - \left(\omega_{u1}+\omega_{d1}\right)\bar{u}_1 + \left(\varepsilon_u+\varepsilon_d\right)\bar{u}_m - \delta_u u_{um} - \delta_d u_{dm}\right]$$

Mass flux Entrainment Detrainment

## Meridional momentum equation

$$\left.\frac{\Delta\bar{v}}{\Delta t}\right|_{CONV} = \frac{1}{\Delta p}\left[\left(\omega_{u2}+\omega_{d2}\right)\bar{v}_2 - \left(\omega_{u1}+\omega_{d1}\right)\bar{v}_1 + \left(\varepsilon_u+\varepsilon_d\right)\bar{v}_m - \delta_u v_{um} - \delta_d v_{dm}\right]$$

# Mass flux and cloud momentum

From continuity

$$\boxed{\frac{\partial M_C}{\partial z} = E - D} \longrightarrow \boxed{-\rho g \frac{\partial M_C}{\partial p} = E - D}$$

-- Updrafts

$$\boxed{-\rho g \frac{\partial M_u u_u}{\partial p} = E_u \overline{u} - D_u u_u}$$

$$\boxed{-\rho g \frac{\partial M_u v_u}{\partial p} = E_u \overline{v} - D_u v_u}$$

-- Downdrafts

$$\boxed{\rho g \frac{\partial M_d u_d}{\partial p} = E_d \overline{u} - D_d u_d}$$

$$\boxed{\rho g \frac{\partial M_d v_d}{\partial p} = E_d \overline{v} - D_d v_d}$$