Eta Model Dynamics

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Part I:

- Approach;

- Gravity-wave coupling/ time differencing;
- Horizontal advection:
- Energy transformations;
- Nonhydrostatic effects

“Philosophy” of the Eta numerical design:

“Arakawa approach”

Attention focused
on the physical properties
of the finite difference analog
of the continuous equations

- Formal, Taylor series type accuracy:
  not emphasized;
- Help not expected from merely increase
  in resolution

"Physical properties . . ." ?

Properties (e.g., kinetic energy, enstrophy) defined using grid point values as model grid box averages /

as opposed to their being values of continuous and differentiable functions at grid points

(Note "physics": done on grid boxes ! !)

Arakawa, at early times:

- Conservation of energy and enstrophy;
- Avoidance of computational modes;
- Dispersion and phase speed;
- . . .

Akio Arakawa:

Design schemes so as to emulate as much as possible physically important features of the continuous system !

Understand/ solve issues by looking at schemes for the minimal set of terms that describe the problem

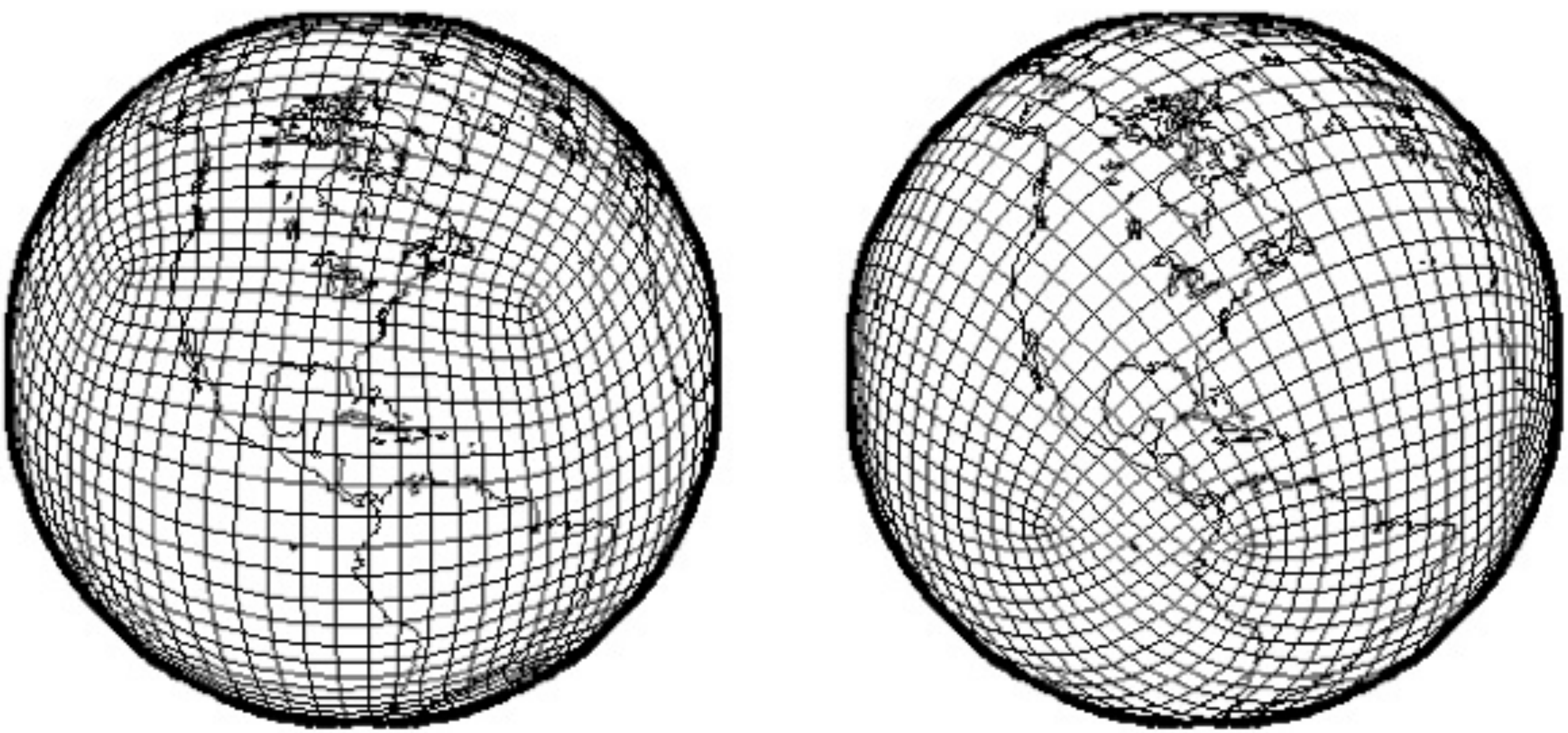
Akio Arakawa:

The Eta (as mostly used up to now) is a regional model:

Lateral boundary conditions (LBCs) are needed (to be briefly summarized later)

There is now also a global Eta Model:

Zhang, H., and M. Rancic: 2007: A global Eta model on quasi-uniform grids. *Quart. J. Roy. Meteor. Soc.*, **133**, 517-528.



Eta dynamics: What is being done ?

- Gravity wave terms, on the B/E grid: forward-backward scheme that
(1) avoids the time computational mode of the leapfrog scheme, and is neutral with time steps twice leapfrog;
(2) modified to enable propagation of a height point perturbation to its nearest-neighbor height points/ suppress space computational mode;

- Split-explicit time differencing (very efficient);

- Horizontal advection scheme that conserves energy and C-grid enstrophy, on the B/E grid, in space differencing (Janjić 1984);

- Conservation of energy in transformations between the kinetic and potential energy, in space differencing;

- Nonhydrostatic option;

- The eta vertical coordinate, ensuring hydrostatically consistent calculation of the pressure gradient ("second") term of the pressure-gradient force (PGF);

- Finite-volume vertical advection of dynamic variables (**v**, T)

- Gravity wave (gravity-inertia wave) scheme

Linearized shallow-water equations:

The forward-backward scheme:

(Richtmyer ?)

$$u^{n+1} = u^n - g\Delta t\, \delta_x h^{n+1},$$

$$v^{n+1} = v^n - g\Delta t\, \delta_y h^{n+1},$$

$$h^{n+1} = h^n - H\Delta t\,(\delta_x u + \delta_y v)^n.$$

Stable, and neutral, for time steps twice those of the leapfrog scheme;

No computational mode

Coriolis terms: trapezoidal scheme

$$u^{n+1} = \ldots \quad + \tfrac{1}{2} f \Delta t\,(v^n + v^{n+1})$$

$$v^{n+1} = \ldots \quad - \tfrac{1}{2} f \Delta t\,(u^n + u^{n+1})$$

Unconditionally neutral

(Fischer, MWR, 1965)

Elimination of u,v from pure gravity-wave system leads to the wave equation; in 1D, for simplicity, (5.6):

(From Mesinger, Arakawa, 1976)

$$\frac{\partial^2 h}{\partial t^2} - gH \frac{\partial^2 h}{\partial x^2} = 0. \qquad (5.6)$$

We can perform the same elimination for each of the finite difference schemes.

The forward-backward and space-centered approximation to (5.5) is

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} + g \frac{h_{j+1}^n - h_{j-1}^n}{2\Delta x} = 0,$$
$$\frac{h_j^{n+1} - h_j^n}{\Delta t} + H \frac{u_{j+1}^{n+1} - u_{j-1}^{n+1}}{2\Delta x} = 0, \qquad (5.7)$$

We now substract from the second of these equations an analogous equation for time level $n-1$ instead of n, divide the resulting equation by Δt, and, finally, eliminate all u values from it using the first of Eqs. (5.7), written for space points $j+1$ and $j-1$ instead of j. We obtain

$$\frac{h_j^{n+1} - 2h_j^n + h_j^{n-1}}{(\Delta t)^2} - gH \frac{h_{j+2}^n - 2h_j^n + h_{j-2}^n}{(2\Delta x)^2} = 0. \qquad (5.8)$$

This is a finite difference analogue of the wave equation (5.6). Note that although each of the two equations (5.7) is only of the first order of accuracy in time, the wave equation analogue equivalent to (5.7) is seen to be of the second order of accuracy.

If we use a leapfrog and space-centered approximation to (5.5), and follow an elimination procedure like that used in deriving (5.8), we obtain

$$\frac{h_j^{n+1} - 2h_j^{n-1} + h_j^{n-3}}{(2\Delta t)^2} - gH \frac{h_{j+2}^{n-1} - 2h_j^{n-1} + h_{j-2}^{n-1}}{(2\Delta x)^2} = 0. \tag{5.9}$$

This also is an analogue to the wave equation (5.6) of second-order accuracy. However, in (5.8) the second time derivative was approximated using values at three consecutive time levels; in (5.9) it is approximated by values at every second time level only, that is, at time intervals $2\Delta t$. Thus, with the leapfrog scheme, as far as the pure gravity wave terms are concerned, we are carrying out two independent integrations at the same time – no wonder it takes twice the computer time to do this !!!

Moving back to 2D: a choice of space grid is needed

Arakawa 1997

Reviews of various discretization methods applied to atmospheric models include Mesinger and Arakawa (1976), GARP (1979), ECMWF (1984), WMO (1984), Arakawa (1988) and Bourke (1988) for finite-difference, finite-element and spectral methods and Staniforth and Côté (1991) for the semi-Lagrangian method.

7.2 Horizontal computational mode and distortion of dispersion relations

Among problems in discretizing the basic governing equations, computational modes and computational distortion of the dispersion relations in a discrete system require special attention in data assimilation. Here a computational mode refers to a mode in the solution of discrete equations that has no counterpart in the solution of the original continuous equations. The concept of the order of accuracy, therefore, which is based on the Taylor expansion of the residual when the solution of the continuous system is substituted into the discrete system, is not relevant for the existence or non-existence of a computational mode.

geostrophic ajustm.:

Arakawa, dynamics:

- geostrophic adjustment
- simulation of slow, quasi-geostrophic motions

"the green book"

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x} + fv,$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial h}{\partial y} - fu,$$

$$\frac{\partial h}{\partial t} = -H\,\nabla\cdot\mathbf{v}$$

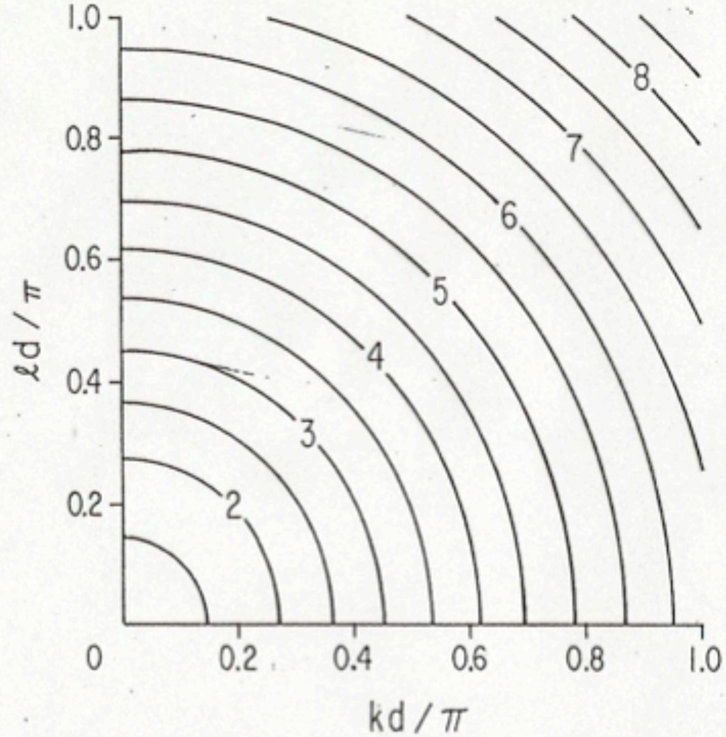


FIG. 9. Contours of the (nondimensional) frequency as a function of the (nondimensional) horizontal wave numbers for the differential shallow water equation for $\lambda/d = 2$, presented for comparison with Fig. 8.

$$\lambda \equiv \sqrt{gH}/f$$

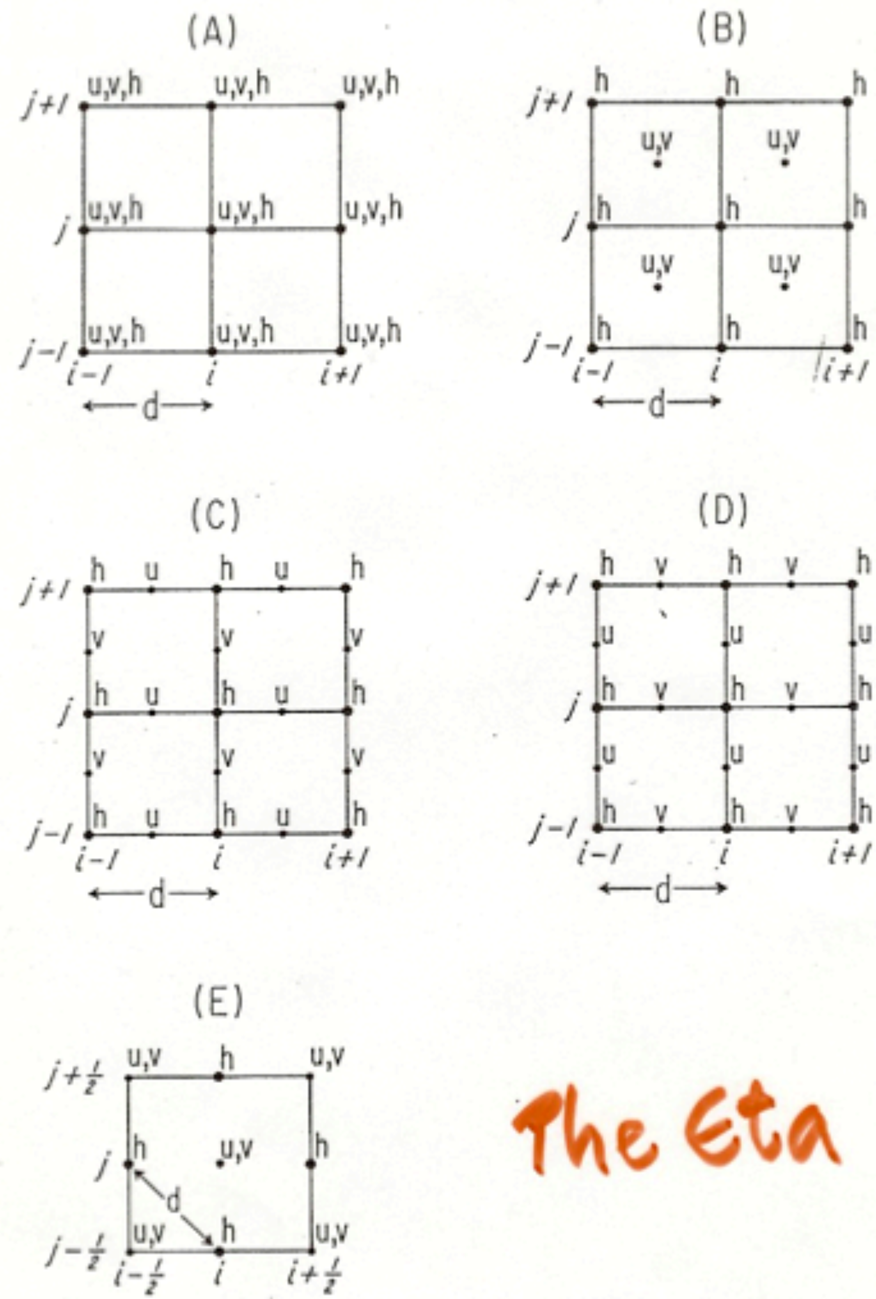
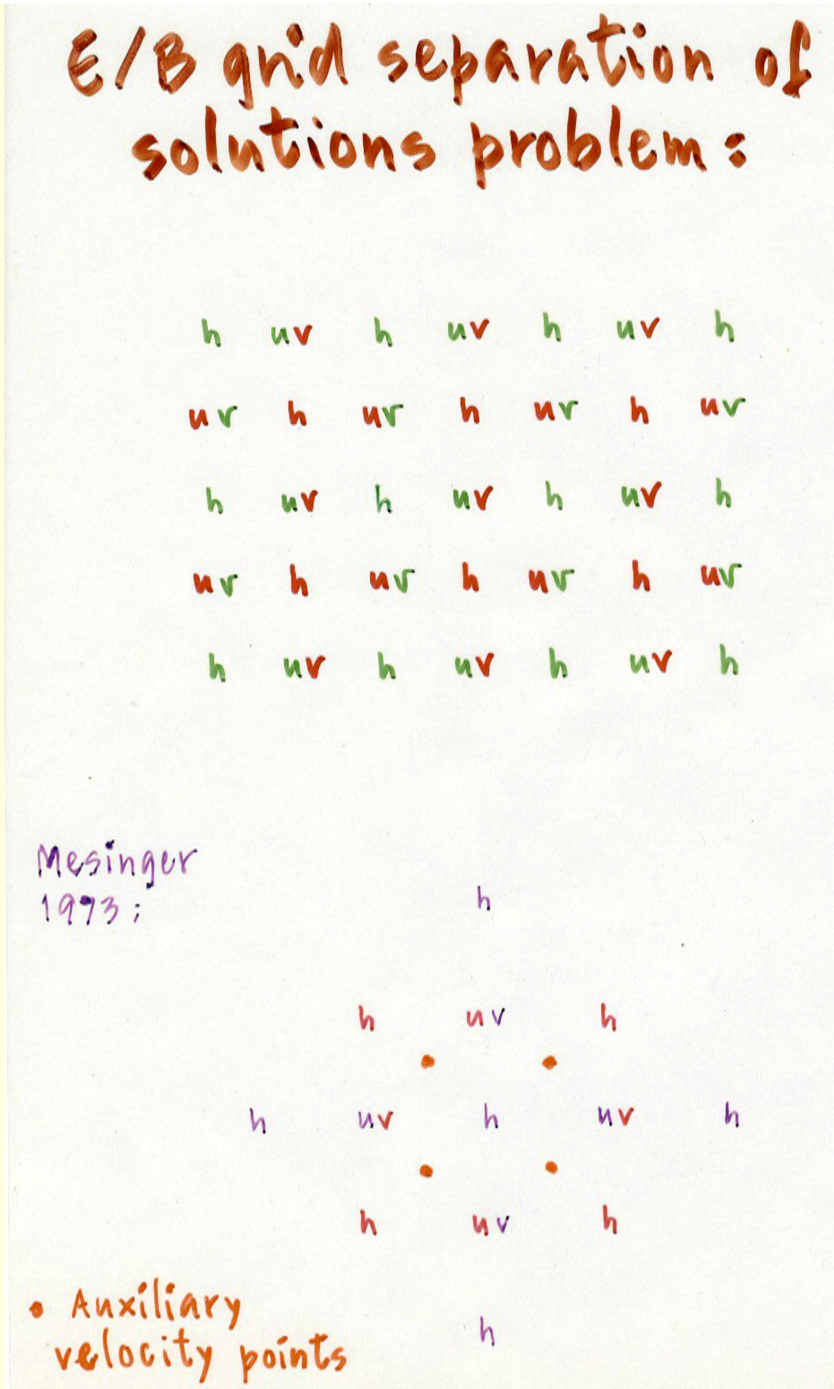


FIG. 3. Spatial distributions of the dependent variables on a square grid.

Note:

E grid is same as B, but rotated 45°. Thus, often: E/B, or B/E



(Two C-subgrids)

"The modification"

Pointed out (1973) that divergence equation can be used just as well; result is the same as when using the auxiliary velocity points

The method, 1973, applied to a number of time differencing schemes;

In Mesinger 1974:

applied to the "forward-backward" scheme

Back to "modification", gravity wave terms only:

on the lattice separation problem. If, for example, the forward–backward time scheme is used, with the momentum equation integrated forward,

$$u^{n+1} = u^n - g\Delta t \delta_x h^n, \qquad v^{n+1} = v^n - g\Delta t \delta_y h^n, \tag{2}$$

instead of

$$h^{n+1} = h^n - H\Delta t\left[(\delta_x u + \delta_y v) - g\Delta t \nabla_+^2 h\right]^n, \tag{3}$$

the method results in the continuity equation (Mesinger, 1974):

$$h^{n+1} = h^n - H\Delta t\left[(\delta_x u + \delta_y v) - g\Delta t\left(\frac{3}{4}\nabla_+^2 h + \frac{1}{4}\nabla_\times^2 h\right)\right]^n. \tag{4}$$

Single-point perturbation spreads to both *h* and *h* points !

Extension to 3D: Janjić, Contrib. Atmos. Phys., 1979

Eq. (4) (momentum eq. forward):

Following a pulse perturbation (height increase) at the initial time, at time level 1 increase in height occurs at four nearest points equal to 2/3 of the increase which occurs in four second nearest points.

This is not ideal, but is a considerable improvement over the situation with no change at the four nearest height points !

In the code: continuity eq. is integrated forward.

"Historic reasons". With this order, at time level 1 at the four second nearest points a decrease occurs, in the amount of 1/2 of the increase at the four nearest points !

Might well be worse? However:

Experiments made, doing 48 h forecasts, with full physics, at two places, comparing

continuity eq. forward, vs momentum eq. forward

No visible difference ! (Why?)

Just published

Mesinger, F., and J. Popovic, 2010: Forward–backward scheme on the B/E grid modified to suppress lattice separation: the two versions, and any impact of the choice made? *Meteor. Atmos. Phys.*, **108**, 1-8, DOI 10.1007/s00703-010-0080-1.

Impact of "modification":

upper panel, used
lower panel, not used

- **Figure 8** Sea level pressure, 00 GMT 24 August 1975, 24 hr forecast with variable boundary conditions. Above: with w = .25; below: with w = 0.

Time differencing sequence ("splitting" is used):

Adjustment stage: cont. eq. forward, momentum backward

(the other way around in the Global Eta)

Vertical advection over 2 adj. time steps

Horizontal diffusion;

Repeat (except no vertical advection now, since it is done for two time steps)

Horizontal advection over 2 adjustment time steps

(first forward then off-centered scheme, approx. neutral);

Some physics calls;

Repeat all of the above;

More physics calls;

. . . . .

Time differencing: split explicit

A variety of time steps:

time

Cont. eq.

1 3 6

2 4 . . . .

"forward-backward" sch.

Mom. eq. (adj. terms, incl. Cor. f. implicit)

5 . . .

Horiz. adv. (~ Heun)

. . .

Some physics
(incl. convection)

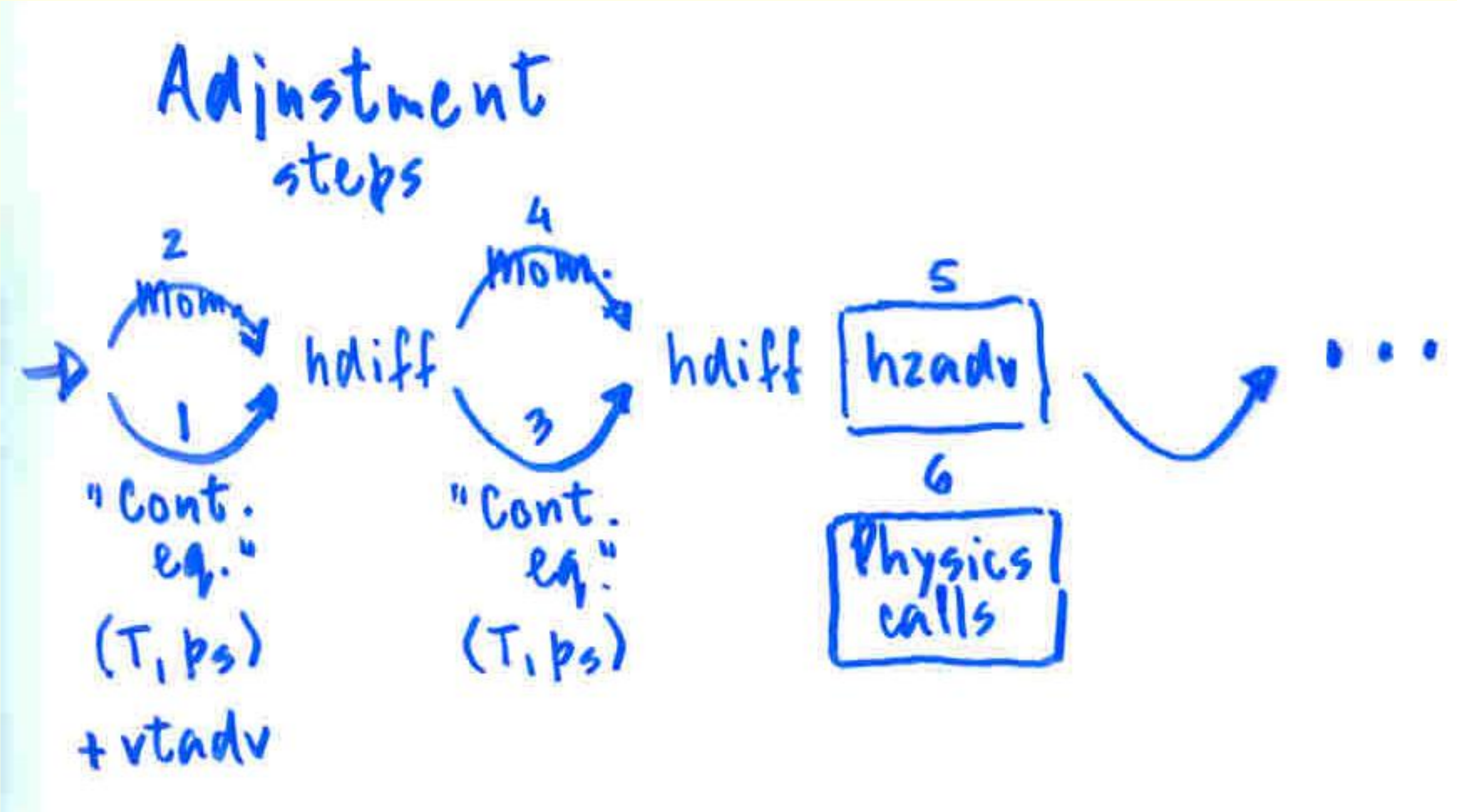
More physics

:

F. Mesinger

However:

"horizontal diffusion" following each forward-backward step:



Adj. step splitting used:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -f\mathbf{k} \times \mathbf{v} - g\nabla h,$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = 0. \qquad (1)$$

is replaced by

$$\frac{\partial \mathbf{v}}{\partial t} = -f\mathbf{k} \times \mathbf{v} - g\nabla h,$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = 0. \qquad (2)$$

as the "adjustment step",

and

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = 0, \qquad (3)$$

as the "advection step"

Note that height advection $\mathbf{v} \cdot \nabla h$ (corresponding to pressure in 3D case) is carried in the adjustment step (or, stage), even though it represents advection!

This is a necessary, but not sufficient, condition for energy conservation in time differencing in the energy transformation ("ωα") term (transformation between potential and kinetic energy). Splitting however, as above, makes exact conservation of energy in time differencing not possible (amendment to Janjic et al. 1995, slides that follow). Energy conservation in the Eta, in transformation between potential and kinetic energy is achieved in space differencing.

Time differencing in the Eta: two steps of (2) are followed by one, over 2Δt, step of (3).

How is this figured out?

To achieve energy conservation in time differencing one needs to replicate what happens in the continuous case. Energy conservation in the continuous case, still, for simplicity, shallow water eqs.:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -f\,\mathbf{k} \times \mathbf{v} - g\nabla h, \qquad (1.1)$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{v}) = 0. \qquad (1.2)$$

To get the kinetic energy eq., multiply (1.1) by $h\,\mathbf{v}$, multiply (1.2) by $\frac{1}{2}\mathbf{v} \cdot \mathbf{v}$, and add,

$$\frac{\partial}{\partial t}\frac{1}{2}h\mathbf{v} \cdot \mathbf{v} + h(\mathbf{v} \cdot \nabla)\frac{1}{2}\mathbf{v} \cdot \mathbf{v} + \frac{1}{2}\mathbf{v} \cdot \mathbf{v}\nabla \cdot (h\mathbf{v}) = -gh\mathbf{v} \cdot \nabla h \qquad (4)$$

For the potential energy eq., multiply (1.2) by *gh*,

$$\frac{\partial}{\partial t}\frac{1}{2}gh^2 + gh\nabla \cdot (h\mathbf{v}) = 0 \qquad (5)$$

Adding (4) and (5) we obtain

$$\frac{\partial}{\partial t}\left(\frac{1}{2}h\mathbf{v} \cdot \mathbf{v} + \frac{1}{2}gh^2\right) + \nabla \cdot \left(\frac{1}{2}\mathbf{v} \cdot \mathbf{v}\, h\mathbf{v}\right) + \nabla \cdot (gh^2\mathbf{v}) = 0. \qquad (6)$$

Thus, the total energy in a closed domain is conserved

For conservation *in time differencing* terms that went into one and the other divergence term have to be available at the same time;

- Kinetic energy in horizontal advection (the 1st divergence term of (6)):

Formed of contributions of horizontal advection of **v** in (1.1), and mass divergence in (1.2)
Not available at the same time with the split-explicit approach; cannot be done;

- Energy in transformations potential to kinetic (the 2nd divergence term):

Formed of the advection of *h* term on the right side of (4), coming from the pressure-gradient force, and the mass divergence term of (5), coming from the continuity eq.;

Both are done in the adjustment stage with the splitting as in (2) and (3);
cancellation is thus possible if the two are done at the same time

However: they are done separately with the forward-backward scheme;

Thus, with the forward-backward scheme cannot be done*;

Time steps used for the adjustment stage very small; not considered a serious weakness (Eta at 10 km resolution is typically using adjustment time step of 20 s)

* Reference for which this is an update:

Janjic, Z. I., F. Mesinger, and T. L. Black, 1995: The pressure advection term and additive splitting in split-explicit models. *Quart. J. Roy. Meteor. Soc.*, **121**, No. 524, 953-957.

- Horizontal advection

The famous **Arakawa horizontal advection scheme**:

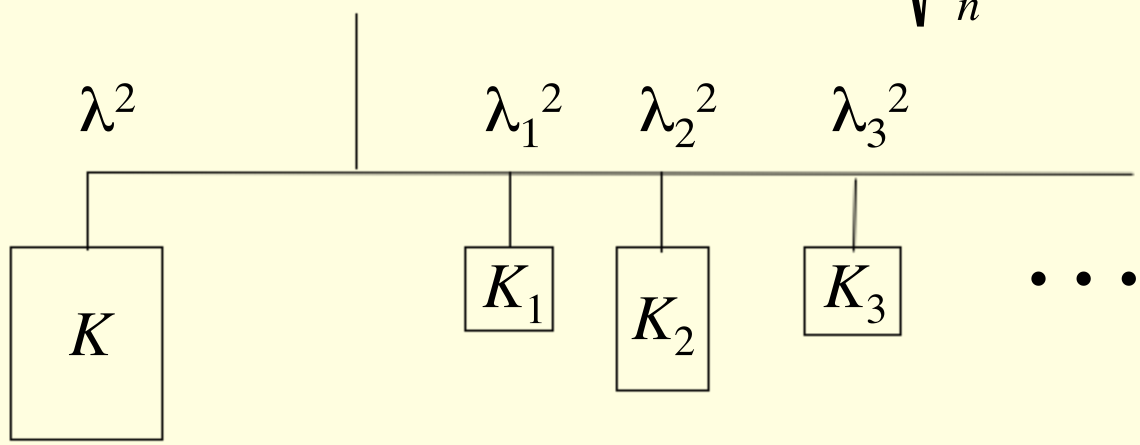
For two-dimensional and nondivergent flow:

One obtains[*], average "enstrophy"=

$$\frac{1}{2}\overline{\zeta^2} = \sum_n \lambda_n{}^2 K_n = \text{const}$$

Define average wavenumber as $\lambda = \sqrt{\sum_n \lambda_n{}^2 K_n / \sum_n K_n}$

Thus:



([*]Fjørtoft 1953, in Mesinger, Arakawa 1976; Charney 1966)

From the preceding slide: $$\lambda^2 \sum_n K_n = \sum_n \lambda_n{}^2 K_n$$

Thus, if one conserves analogs of average enstrophy

$$\frac{1}{2}\overline{\zeta^2} = \sum_n \lambda_n{}^2 K_n$$

and of total kinetic energy $$\sum_n K_n$$

analog of the average wavenumber will also be conserved !!!

Arakawa 1966:
Discovered a way to reproduce this feature for the vorticity equation

Primitive equations ?

Note:

E grid is same as B, but rotated 45°. Thus, often: E/B, or B/E

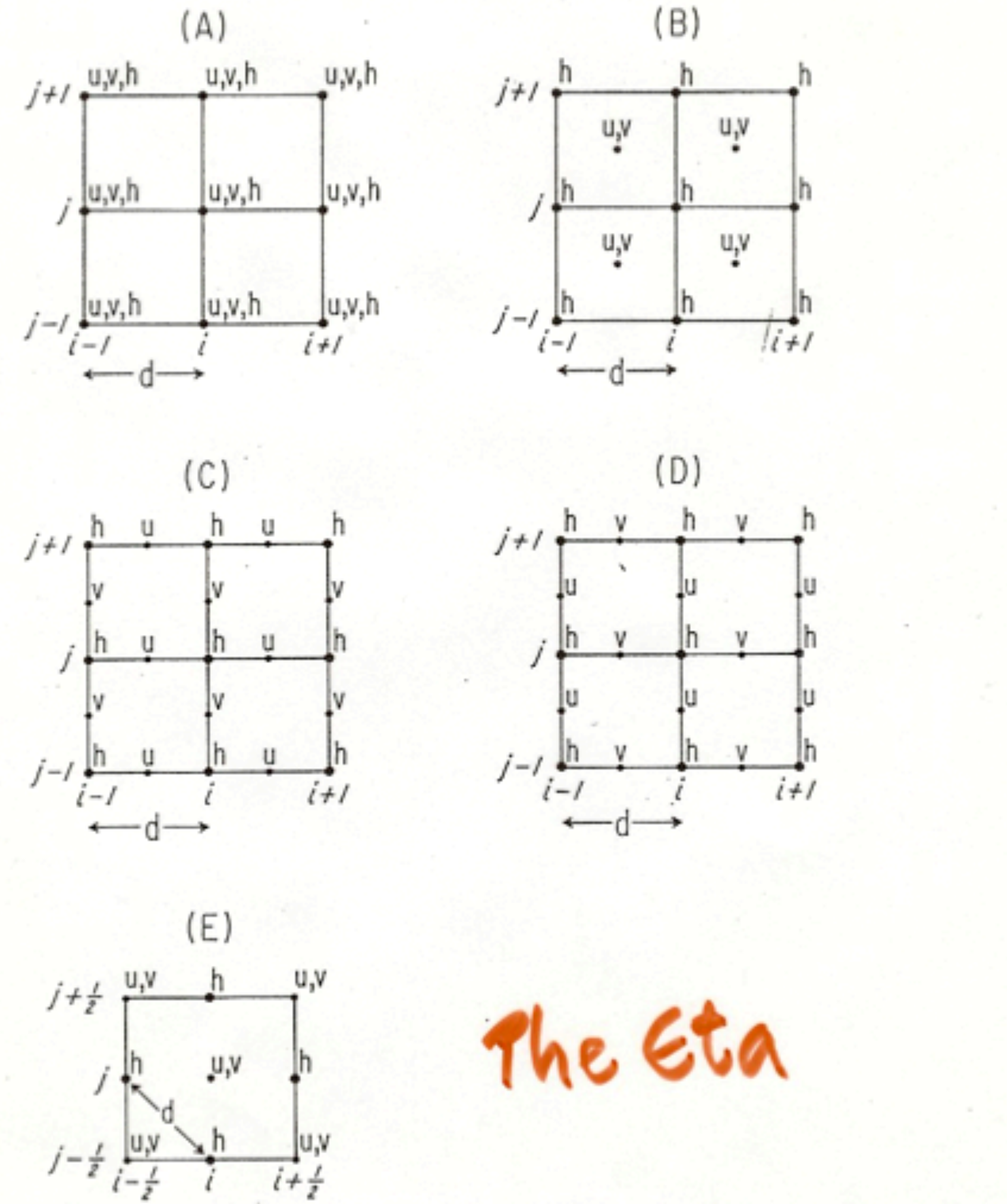
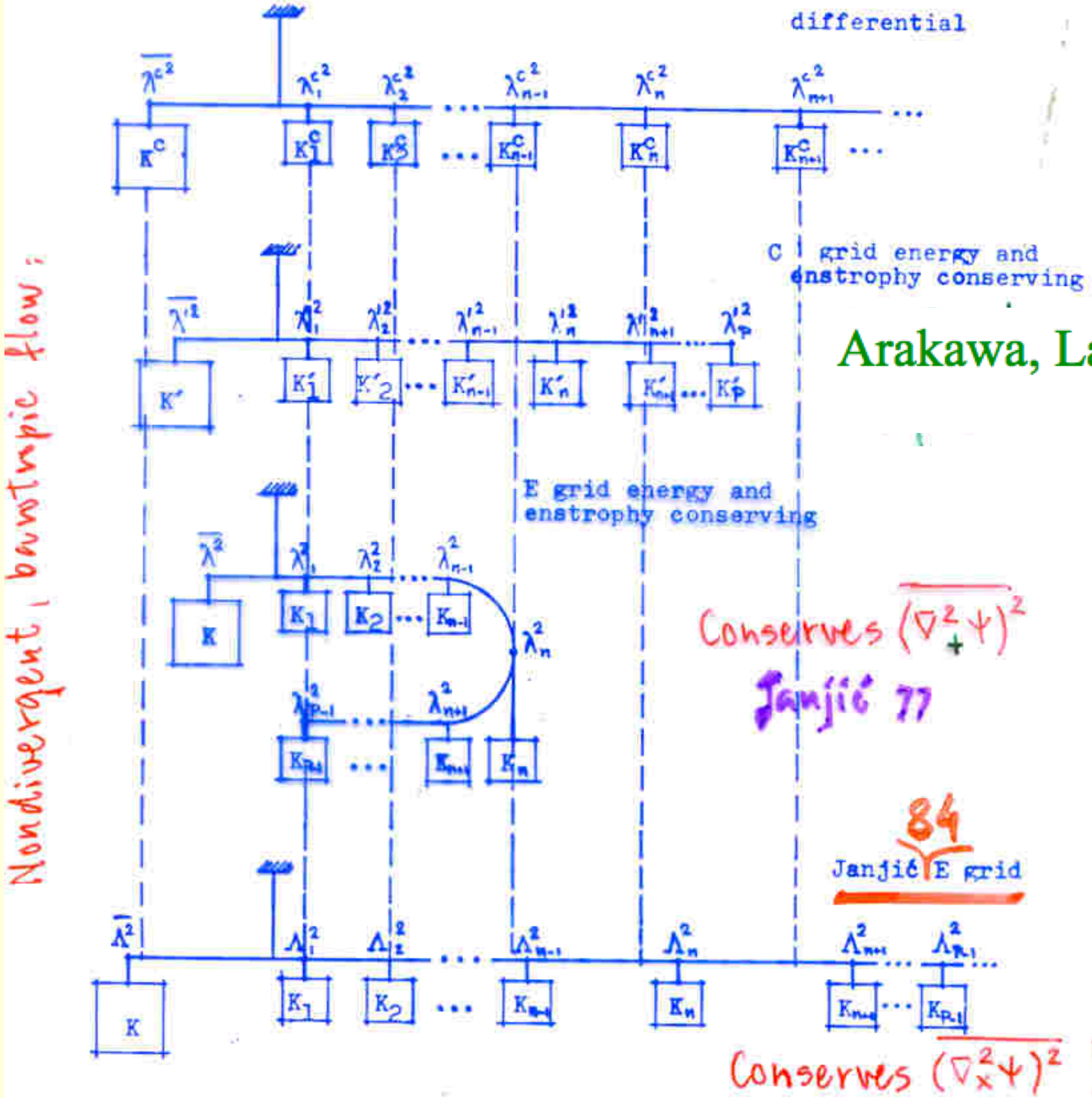


FIG. 3. Spatial distributions of the dependent variables on a square grid.

From ECMWF Seminar 1983:

The horizontal advection scheme:



Arakawa, Lamb (1977)

Fig. 3.12. Mechanical analogies of the constraints imposed on the non-linear energy cascade in the continuous case, in the case of the C-grid energy and enstrophy conserving scheme, in the case of the E-grid energy and enstrophy conserving scheme, and in the case of the scheme due to Janjić (1984).

(ECMWF Seminar '83)

Horizontal advection: conserve enstrophy ($\Sigma \frac{1}{2}\zeta^2$) and kinetic energy for nondivergent barotropic part of the flow!

MWR, 1984

ZAVIŠA I. JANJIĆ

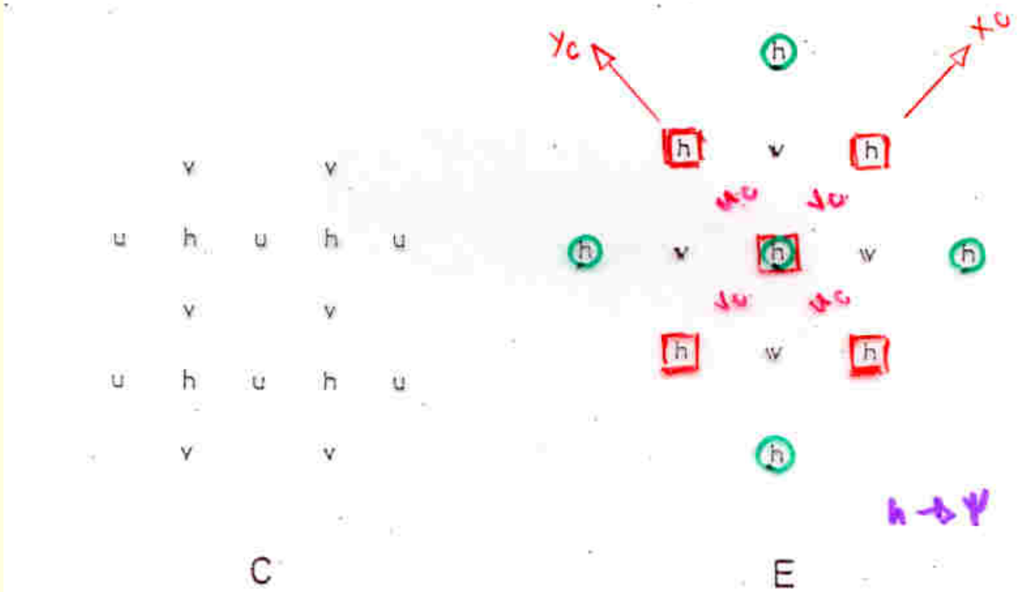


FIG. 1. Distributions of variables over grid points C and E.

Problem: ζ_E defined by simple differencing of the E grid u, v components eq. $\nabla_+^2 \psi$, using ψ values at ○ points.

ζ_C, defined by differencing u_C, v_C, is equal to $\nabla_\times^2 \psi$, using ψ values at □ points!

Janjic 1984:

- *Arakawa-Lamb C grid* scheme written in terms of u_C, v_C;
- write in terms of stream function values (at h points of the right hand plot);
- these same stream function values (square boxed in the plot) can now be transformed to u_E, v_E

Janjić adv. scheme

	Conserves: No inter. bnd.	Conserves: With int. bnd.
Nondivergent part of the flow		
C-grid enstrophy	✓	✓
vorticity	✓	✓
rotat. energy	✓	✓
Divergent part incl.		
mass	✓	✓
E-grid kin. energy	✓	✓
momentum	✓	O
Passive quantity (T, q, q2, hor. adv.)		
1st moment	✓	✓
2nd moment	✓	✓

From Janjic, MWR 1984: Initial field wavenumbers 1-3, but mostly 2;

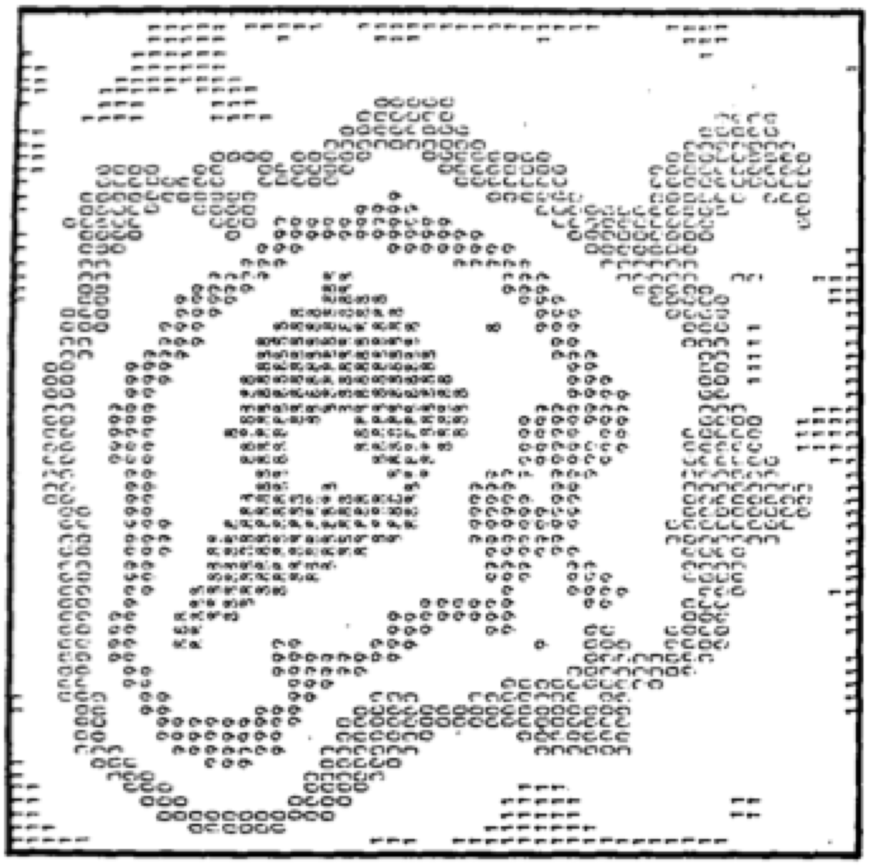


FIG. 13. Height field after 10 000 time steps in the control experiment. The shading interval is 160 m.

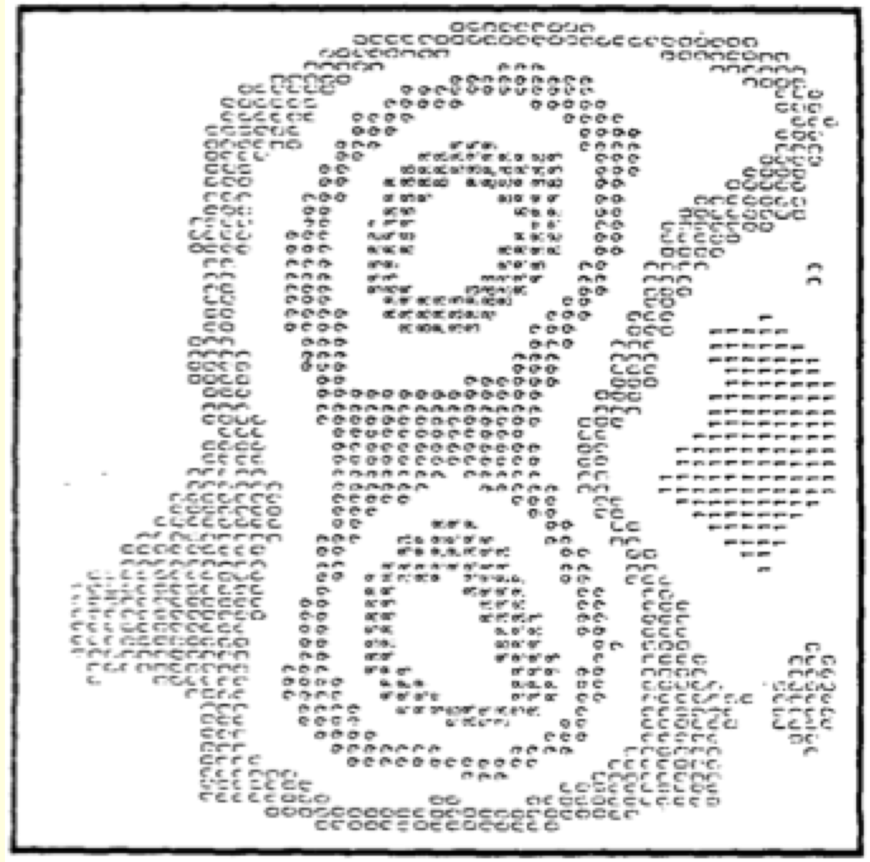


FIG. 12. Height field after 10 000 time steps in the main experiment. The shading interval is 160 m.

Left, Janjic 1977 – inaccurate (bent) analog of the Charney energy scale; Right, Janjic 1984 – a straight scale analog: no systematic transport to small scales (noise !), average wavenumber well maintained

• Conservation of energy in transformation kinetic to potential, in space differencing

- Evaluate generation of kinetic energy over the model's **v** points;
- Convert from the sum over **v** to a sum over T points;
- Identify the generation of potential energy terms in the thermodynamic equation, use appropriate terms from above

(2D: Mesinger 1984, reproduced and slightly expanded in

Mesinger, F., and Z. I. Janjic, 1985: Problems and numerical methods of the incorporation of mountains in atmospheric models. In: *Large-Scale Computations in Fluid Mechanics*, B. E. Engquist, S. Osher, and R. C. J. Somerville, Eds. Lectures in Applied Mathematics, Vol. 22, 81-120.

Downloadable in a bit earlier form at

http://www.ecmwf.int/publications/library/do/references/list/16111

3D: Dushka Zupanski in Mesinger et al. 1988)

Nonhydrostatic option (a switch available),

Janjic et al. 2001:

$$\left(\frac{\partial w}{\partial t}\right)^{\tau+1/2} \rightarrow \frac{w^{\tau+1} - w^{\tau}}{\Delta t}$$

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